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THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

Edited by

GEORGE E. HALE
Mount Wilson Observatory of the Carnegie Institution of Washington

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APRIL 1929

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THE STARK EFFECT IN STELLAR SPECTRA

By OTTO STRUVE

ABSTRACT

The great widths of the hydrogen lines in stellar spectra are probably due in part to *mol-electric Stark broadening*. An estimate of the average *intensity* of the *intramolecular fields* gives 10^3–10^4 volts per centimeter. *Such fields*, if really present, *must affect* also the *atoms of elements other than H*.

Measurements of line-widths in B- and A-type stars show that narrow and wide lines occur in all spectral subdivisions. The narrow lines are usually strong in the center, and the wide lines are shallow. The total energy absorbed does not vary much within any one subdivision. This shows that the *mechanism of broadening is not identical* with the one due to *abundance* which has given good results in the later types. It is suggested that the *broadening in early types* is due in part to *Stark effect*, to *axial rotation*, and to *abundance*.

The *general appearance* of the lines *agrees, qualitatively*, with the requirement of the *theory* that the *H lines* should be *broadest*, that *He* and the other light elements should also be *widened appreciably*, but that the *widening of* the lines of the *heavier elements should be small*.

Since the intensity of the intramolecular fields depends upon the partial electron pressure, a *relation* was found connecting *line-width* and *absolute magnitude*. The stars with narrow lines should be more luminous than those with wide lines. This agrees with the well-known luminosity effect discovered by Adams and Joy.

The presence of *electric fields* of the order of 10^4 volts per centimeter should stimulate the appearance of *new lines forbidden* by the *selection principle* of the quantum theory. It is suggested that the *strongest forbidden line of He* belonging to the "near-diffuse" series (λ 4470) may be *identical* with a *line of unknown origin* at λ 4470 which is present in many B stars and reaches maximum intensity at about the same point of the spectral sequence as do the normal *He* lines.

The *He* lines are *known* to broaden unsymmetrically in the fluctuating Stark effect. The *expected shifts* have been *estimated* for the more prominent lines and were found to be in *good agreement* with determinations of wave-length by Albrecht, based upon measures of Frost and Adams.

The *Balmer lines should broaden* in the Stark effect in varying proportions. There should be a *marked increase in width* from $H\beta$ to $H\epsilon$. The measures of Elvey show that an increase exists, but its amount is smaller than might be expected. It is suggested that the *theoretical increase* in line-width is partially *balanced* by a *superposed mol-collisional broadening* depending upon the *motion* of the *charged particles*.

The *evidence* so far available *is regarded as favorable* to the idea that the *broadening in early spectral types* is partly due to the *interaction of the various atoms and free electrons* in the reversing layers of the stars.

I. INTRODUCTION

It was suggested by J. Stark[1] in 1906 that the electric fields of neighboring ions or free electrons might be expected to produce appreciable perturbations in the electronic vibrations of a radiating atom. The discovery, in 1913, of the resolution of spectral lines into a number of components when subjected to a strong electric field fully supported this view. The amount of resolution was found to be very nearly proportional to the intensity of the field, which in turn depends upon the strength of the charge and its distance from the radiating atom. In a gas containing many free electrons and positive ions the distances of the charged particles are distributed at random; consequently, the components produced by the individual atoms will not coincide but will blend into a broadened spectral line. The amount and character of this broadening must be closely related to the original Stark effect in a constant field. If, for example, a line splits up into two components, both shifted toward the red, the integrated line must also be broadened toward the red. Stark[2] remarked, however, that in addition to this blended resolution into components there should be superimposed a symmetrical broadening due to the relative motion of the charged particle with respect to the radiating atom. The total effect should thus consist of two factors: the mol-electric broadening and the mol-collisional broadening, the mechanical action of the moving field of force being considered as a collision.

The ideas of Stark have been tested by many investigators in the laboratory, and the results were invariably in good agreement with the hypothesis. It is now generally believed that the Stark effect is responsible for at least a large part of the observed increase in width with increase of pressure; and in the presence of many charged particles this type of broadening becomes far more prominent than that due to the thermal agitation of the atoms, or to the ordinary mechanical collisions between them. It seems that under ordinary laboratory conditions—as, e.g., in a capillary discharge

[1] *Verhandlungen der Deutschen Physikalischen Gesellschaft*, **8**, 109, 1906; *Annalen der Physik*, **21**, 422, 1906.

[2] J. Stark, *Handbuch der Experimental-Physik*, **21**, 399, 1927; *Jahrbuch der Radiologie und Elektronik*, **12**, 349, 1915.

tube—the observed line-widths are fully accounted for[1] by the blended resolution into components, i.e., by that part of the effect which Stark called "mol-electric."

It is obvious that this type of broadening depends exclusively upon the concentration of charged particles in the given gas. It will be sponsored by strong ionization and high density. The first factor is present in the stars, but the pressure in the reversing layers is so low that it is not at once evident whether the Stark effect will play any important part in the broadening of stellar absorption lines.

Probably the first to consider the Stark broadening with respect to stellar spectra was R. de R. Atkinson.[2] He pointed out that, according to Debye,[3] the mean molecular field in a gas where there are 10^{-6} gr. per cubic centimeter of ions is about 10^{4} volts per centimeter and that this field would tend to broaden the spectral emission lines of the photospheric atoms sufficiently to explain the continuous spectrum in giant stars having average densities of 10^{-6} gr. per cubic centimeter.

It was suggested by E. O. Hulburt[4] that the great width of the *H* lines in early spectral types might be due to Stark broadening. His numerical computations gave, it is true, predicted values which are much smaller than the observed ones, but he believed that the discrepancy could be removed by assuming a greater concentration of free electrons, resulting, perhaps, from the high state of ionization of the heavier elements in the hotter stars.

H. N. Russell and J. Q. Stewart[5] derived for the $H\beta$ line broadened by the Stark effect the expression:

$$W = (2.3 \times 10^{5})(p'/T)^{2/3}, \qquad (1)$$

where p' is the pressure of the free electrons in atmospheres. Assuming for the sun $T = 5000°$, $p' = 10^{-5}$, they find $W = 0.4$ A. The

[1] E. O. Hulburt, *Physical Review*, **22**, 24, 1923; Mlle M. Hanot, *Journal de Physique et le Radium*, **9**, 156, 1928.

[2] *Monthly Notices of the Royal Astronomical Society*, **82**, 396, 1922.

[3] *Physikalische Zeitschrift*, **20**, 160, 1919.

[4] *Astrophysical Journal*, **59**, 177, 1924; *Journal of the Franklin Institute*, **201**, 777, 1926.

[5] *Astrophysical Journal*, **59**, 204, 1924.

observed value is about 0.6 A. Assuming for α Lyrae $T = 10{,}000°$ and $p' = 10^{-4}$, they find $W = 1.1$ A, the observed value being 20 A. The agreement for the sun is satisfactory, but for α Lyrae the discrepancy is considerable, although not hopeless. An uncertainty in p' corresponding to a factor of 10 would not be unreasonable, but, what is more important, the total intensities of the H lines in α Lyrae is greater than in the sun; consequently the lines will appear relatively broader in α Lyrae because of the greater abundance of the atoms capable of absorbing them.

A very recent discussion of the Stark broadening for the H lines is by M. Vasnecov.[1] He obtains for the *Halbwertsbreite*, or "half-width," δ, which corresponds to a point on the line-contour where $I = \frac{I_0}{2}$:

$$\delta = (3.45 \times 10^5) f \cdot \lambda_0^2 \cdot (\chi \cdot p \cdot T^{-1})^{2/3} . \qquad (2)$$

χ is the concentration of the ionized atoms, p is the pressure, and f is the mean value for the given line of Sommerfeld's[2] expression depending upon the quantum numbers:

$$f = \{(n_2 - n_1)(n_1 + n_2 + n_3) - (k_2 - k_1)(k_1 + k_2 + k_3)\} . \qquad (3)$$

The actual width, according to Vasnecov, is the value $\Delta = 2(\lambda - \lambda_0)$, corresponding to $I = 0.1 I_0$, and for the H lines

$$\Delta = 12\delta .$$

Assuming for the sun $T = 6000°$ and $P = 10^{-5}$, and for αLyrae $T = 10{,}000°$ and $P = 10^{-4}$, Vasnecov's tables give:

$$\Delta \text{ (sun)} = 1 \text{ A} ,$$
$$\Delta (\alpha \text{ Lyrae}) = 42 \text{ A} .$$

These results are in good agreement with the observations.

Recently the whole subject has been reviewed by A. S. Eddington.[3] The evidence must be regarded as favorable to the Stark effect. He believes that even if the fluctuating Stark effect should not

[1] *Vestnik Kral. Ces. Spol. Nauk*, 2, 1927.

[2] *Atomic Structure and Spectral Lines* (Eng. ed.), p. 286, 1923.

[3] *Der innere Aufbau der Sterne*, p. 443, 1928.

prove to be sufficient to explain the observed line-widths, it is likely that the broadening would be increased through the rapid motion of the charged particles. This idea, similar to that of Stark, attributes part of the broadening to the velocity of the particles. We must keep this possibility in view, but shall first proceed according to the point of view of Hulburt, and consider only the ordinary mol-electric broadening.

If the great width of the *H* lines in early type stars is really due to mol-electric broadening, we are led to assume that the average intramolecular field in the reversing layers is quite strong—of the order of 10^3–10^4 volts per centimeter—and that the number of atoms subjected to fields of 10^5 volts per centimeter is still sufficiently great to produce a measurable amount of absorption. Fields of such strength would affect also the atoms of elements other than hydrogen, and it is our purpose to test the expected manifestations of such fields.

II. GENERAL NATURE OF STARK EFFECT

We shall summarize briefly the various effects characteristic of the Stark broadening of spectral lines.[1]

a) The amount of broadening should be roughly proportional to the degree of resolution in a constant field, although because of the motion of the particles there may be superposed a symmetrical spreading-out, similar in character to the mol-collisional broadening.

b) Since the resolution into components is frequently not symmetrical, the broadening should in such cases be accompanied by a pressure shift. The *H* lines remain always symmetrical,[2] but beginning with *He* such shifts are very common. They are complicated by blending with new lines which occasionally appear very close to the parent-lines.

c) Under the stimulus of the electric field new lines appear in the spectrum which are normally forbidden by the selection principle.

[1] For a detailed description of the Stark effect see: Stark, *Handbuch der Experimental-Physik*, **21**, 399, 1927; W. Steubing, *Handbuch der physikalischen Optik*, **2**, 683, 1927; G. S. Fulcher, *Astrophysical Journal*, **41**, 359, 1915.

[2] It should be added, for completeness, that "in hydrogen, as well as in helium, the line-groups presenting a symmetrical appearance as a whole are in reality composed of highly asymmetric individual patterns" (J. Stuart Foster, *Proceedings of the Royal Society*, A, **117**, 137, 1927).

Lines violating the rule of azimuthal-quantum number, $\Delta k = \pm 1$, frequently appear, and in strong fields some of them may become as strong as the original lines.[1]

d) The Stark effect is most pronounced for elements of low atomic number and decreases for the heavier elements. A rough idea of the relative susceptibility of various elements can be gathered from Table I, which gives an estimate of the maximum displacement observed in the stronger lines of the region of wave-lengths covered by stellar spectrographs. The values given refer to a field of about

TABLE I

Element	Atom. Wt.	Max. Displ. in A	Element	Atom. Wt.	Max. Displ. in A
H	1	10	*Mg*	12	1
He	2	5	*Al*	13	0
He^+	2	2	*A*	18	0.1
Li	3	3	*Ca*	20	1
N	7	4	*Fe*	26	0.5
N^+	7	0	*Co*	27	1
O	8	8	*Ni*	28	1
O^+	8	0	*Cu*	29	0.5
Ne	10	3	*Ag*	47	1
Na	11	2	*Au*	79	0.2

30,000 volts per centimeter. This is only a schematic representation, since individual lines of the same element may behave very differently.

e) The effect is most pronounced in the diffuse series of any given element. Next follows the principal series, then the sharp series, and finally the various intercombination series.

f) The effect is smallest for the lowest members of any given series and increases in size as the series limit is approached.

g) For the *H* lines the amount of resolution is proportional to the intensity of the field, *F*, up to about 10^5 volts per centimeter. For stronger fields a quadratic term becomes noticeable. In other elements the quadratic term sets in at much lower values of *F*, but for moderate fields the simplifying assumption

$$W \propto F \qquad (4)$$

[1] W. Grotrian, *Struktur der Materie*, 7, Part I, 119, 1928.

remains a good approximation. The broadest observed H lines are about 40–60 A wide, corresponding to maximum fields of something more than 10^5 volts per centimeter. It is probable that the average fields in the reversing layers of the stars never exceed 10^5 volts per centimeter or about 330 c.g.s. electrostatic units.

h) The theory of the Stark effect has been worked out by A. Sommerfeld[1] and by P. Epstein[2] for hydrogen and similar atomic models, such as He^+. The distance of any one component from its normal position is

$$\Delta\lambda = \frac{3h\lambda_0^2 \cdot F}{8\pi^2 mZc} \cdot f \,, \tag{5}$$

where f is given by (3), m is the mass of the electron, Z is the nuclear charge, h is Planck's constant, and c is the velocity of light. The average value of f to be used to compute the effect of broadening has been given by Vasnecov:

$$f_{H\alpha} = 4; f_{H\beta} = 10; f_{H\gamma} = 18; f_{H\delta} = 28 \,.$$

i) Formula (5) predicts that He^+, where $Z = 2e$, should be broadened less than H. This has actually been observed. From analogy it seems probable that, in general, ionized atoms will show less Stark effect than neutral atoms. This has indeed been verified in several cases (e.g., N^+ and O^+). However, T. R. Merton[3] has pointed out that occasionally He^+ λ 4686 is the broadest line visible in the *He* spectrum, and has given a satisfactory explanation for this phenomenon.

III. LINE-WIDTHS IN STELLAR SPECTRA

Turning our attention to the interpretation of the observed line-widths in stellar spectra, we note that almost certainly a number of entirely different mechanisms are at work. Recent theoretical work by J. Q. Stewart[4] and by Unsöld,[5] based upon the classical electron theory, has shown that the width of a line must increase as the square

[1] *Atomic Structure and Spectral Lines*, p. 286, 1923.

[2] *Annalen der Physik*, **51**, 184, 1916.

[3] *Proceedings of the Royal Society*, A, **95**, 30, 1919.

[4] *Astrophysical Journal*, **59**, 30, 1924.

[5] *Zeitschrift für Physik*, **44**, 793, 1927.

root of the total number of absorbing atoms in the line of sight. Unsöld has shown that this mechanism of broadening satisfactorily explains the contours of many lines in the solar spectrum. Miss C. H. Payne[1] has applied Unsöld's formula to a large number of stars and has found substantial agreement in the later spectral types. The *H* lines, however, proved to be abnormal;[2] they were very much too shallow in the center and too widely spread—a phenomenon which becomes particularly striking in the A, B, and O stars.

According to an investigation by H. N. Russell and Miss C. E. Moore,[3] most of the winged lines in the solar spectrum bear out the argument of Stewart and of Unsöld that "the main cause of widening is shown to be almost entirely the abundance in the solar atmosphere of atoms which are in a condition to absorb the line in question," and "widening in the laboratory shows little correlation with widening in the Sun, and it is probably due to quite different causes." On the other hand, they state that "the widening of the *H* lines in the Sun . . . . may be largely due to the influence of neighboring molecules, since *H* is unusually sensitive to the Stark effect. It is also, of course, peculiarly sensitive to Doppler widening."

While most of the solar lines, including H and K of Ca^+, are widened principally by a mechanism that we may call "abundance-broadening," the *H* lines suggest that another type of mechanism is also present.

IV. LINE-WIDTHS IN STARS OF EARLY TYPE

The peculiarity noted in the *H* lines of the solar spectrum is quite general in stars of early spectral type. I have recently measured and estimated the line-widths (W) and the central intensities (I_0) in a large number of B- and A-type stars. The results for *He* λ 4472 and Mg^+ λ 4481 prove that the dispersion in W as well as in I_0 is considerable within a single spectral subdivision. A summary for λ 4472 in B stars is given in Table II. The central intensities and the line-widths are expressed in arbitrary units, but a calibration of each scale is given at the end of the table.

[1] *Proceedings of the National Academy of Sciences*, **14**, 399, 1928; *Harvard College Observatory Circular*, No. 334, 1928.

[2] Unsöld (*Zeitschrift für Physik*, **46**, 778, 1928) has also shown that the *H* lines in the solar spectrum cannot be represented satisfactorily by his formula. The discrepancy is least for $H\alpha$, and increases for the higher members of the Balmer series.

[3] *Astrophysical Journal*, **63**, 1, 1926.

TABLE II

W	No.	I_o
Type O		
3.3	3	9.0
5.3	3	7.7
9.0	3	3.3
Type Bo		
2.4	5	8.6
4.5	6	7.3
7.3	3	6.1
10.0	2	3.0
Type B1		
1.6	5	10.3
3.3	3	10.0
5.7	4	6.2
10.0	2	3.0
Type B2		
1.0	4	9.5
2.8	4	10.1
4.2	4	7.8
7.0	4	6.2
9.0	4	5.2
Type B3		
1	6	10.3
2	10	8.6
3	12	8.8
4	9	8.0
5	8	7.8
6	10	6.8
7	5	5.6
8	9	6.2
9	8	5.8
10	14	2.7
15	1	1.0
Type B5		
1	2	7.5
2	5	6.2
3	9	4.2
4	12	4.2
5	11	3.8
6	5	5.0
7	8	2.8
9.2	6	3.2
Type B8		
1	2	4.5
2	7	4.6
3	10	4.7
4	20	2.6
5	13	3.1
6	5	1.8
7	4	2.0
8.7	3	2.3
Type B9		
1	2	4.5
2	5	2.8
3	11	2.8
4	4	2.5
5	6	2.6
6	2	2.5
7	1	1.
8	1	1.
9	1	1.
10	1	1.

CALIBRATION OF LINE-WIDTHS

Arb. Un.	A	Arb. Un.	A
1.....	1.2	6.....	2.3
2.....	1.4	7.....	2.5
3.....	1.7	8.....	2.8
4.....	1.9	9.....	3.0
5.....	2.1	10.....	3.2

CALIBRATION OF INTENSITIES

Arb. Un.	Per Cent Absorption	Arb. Un.	Per Cent Absorption
1.....	9	6.....	42
2.....	17	7.....	47
3.....	24	8.....	52
4.....	31	9.....	56
5.....	37	10.....	60

An inspection of the table immediately shows that wide lines are invariably weak in the center, and narrow lines are invariably strong. This in itself suggests a mechanism other than "abundance-broadening."

Let us assume that the intensity of any point on the line-contour is given by an equation of the form

$$I = I_0 e^{-h^2(\lambda-\lambda_0)^2} . \tag{6}$$

The measured width W corresponds to such a value of $(\lambda\text{-}\lambda_0)$ that at this distance from the center of the line the value of I is the smallest perceptible difference in intensity on the background of the continuous spectrum. Putting this equal to I', we have

$$I' = I_0 e^{\frac{-h^2W^2}{4}} ,$$

whence

$$W = \frac{2}{h}\sqrt{\log_e I_0 - \log_e I'} . \tag{7}$$

Thus W is a function not only of h but also of the ratio I_0/I'. If $I' = I_0$, the width is necessarily zero, irrespective of h. For large values of I_0/I', the factor under the root is of little importance, and then $W \sim 1/h$, as is frequently assumed to be the case. But for faint lines the corrective term becomes of considerable importance.

The numerical value of I' depends upon the contrast factor of the photographic plate. Under normal conditions the faintest definitely visible line corresponds to $I_0 = 9$ per cent absorption. The width of such a line may be assumed to be equal to its half-width, giving $I' = 4$ per cent.

The total energy absorbed in a line is, of course,

$$E = I_0 \int_{-\infty}^{+\infty} e^{-h^2(\lambda-\lambda_0)^2} d\lambda = \frac{I_0}{h} \int_{-\infty}^{+\infty} e^{-x^2} dx = \frac{I_0\sqrt{\pi}}{h} = 1.77\frac{I_0}{h} .$$

Substituting h from (7), we get

$$E = \frac{0.88 I_0 W}{\sqrt{\log_e I_0 - \log_e I'}} . \tag{8}$$

I_0 and I' are here expressed in terms of the intensity of the continuous spectrum. If this is known from the law of radiation, E can be expressed in absolute units.

A computation of E by means of formula (8), from the data of Table II, shows that it changes very little with W. For type B_3, where the number of stars is large, there appears to be a slight increase in E with W, but the reality of this change may be doubted in view of the uncertainty of the calibrations of my scales of estimates. It seems that the law governing the broadening in these stars requires approximately:

$$E = \text{Const} . \tag{9}$$

Unsöld's mechanism does not help us here. His formula gives for any line $I_0 = 100$ per cent absorption, and as a rough approximation

$$E \backsim W . \tag{10}$$

It is possible that the observed slight increase of E with W may be due to a superimposed "abundance-broadening," but in view of the uncertainty of my observations this result remains doubtful.

There are a number of broadening mechanisms that approximately satisfy the condition $E = \text{Const}$. Probably the most powerful ones are: (1) axial rotation (or pulsation in Cepheids, expansion in novae); (2) Stark effect.[1] The first mechanism has been discussed in a paper by Shajn and the writer.[2] It is probably present in a measurable degree in many stars, and may attain very large proportions in certain spectroscopic binaries (V Puppis, μ^1 Scorpii, W Ursae Majoris). However, the fact, mentioned first by H. C. Vogel,[3] that the H lines are frequently very much broader than any other lines, makes it practically certain that axial rotation is not the only mechanism responsible for (9). Accordingly, it becomes necessary to assume that the dispersion in W is due to at least three causes:

$$\sigma_{obs} = \sqrt{\sigma^2_{rot} + \sigma^2_{Stark} + \sigma^2_{abun}} \tag{11}$$

[1] It is not certain that the total energy absorbed in a line will remain undisturbed in the Stark effect. Indeed, Stark suggests that the intensity of the diffuse series decreases in favor of the new near-diffuse and almost-sharp series. The question has not been investigated sufficiently to permit definite conclusions. Our assumption that E remains approximately constant is based upon the fact that while the diffuse line λ 4472 gets fainter, the near-diffuse line, λ 4470, gets stronger so that the blend will probably change but little. The energy lost in the almost-sharp series is probably negligible.

[2] *Monthly Notices of the Royal Astronomical Society*, **89**, 222, 1929.

[3] *Astronomische Nachrichten*, **90**, 71, 1877.

The rotational dispersion can be separated, statistically, in short-period spectroscopic binaries. The dispersion due to abundance can be best studied in later spectral types and in lines not greatly influenced by Stark effect. The third mechanism must be investigated by comparing it with the known properties of the Stark effect.

V. ELEMENTS OF DIFFERENT ATOMIC WEIGHT

According to section II(*d*), we may expect that the broadest lines will be those of *H*, closely followed by the lighter elements. The heavier elements should always show fairly narrow lines. Barring all cases of widening due to abundance, this agrees qualitatively with the observations. Next to those of *H*, the broadest lines are doubtless those of *He*, followed by those of the other light elements. The metallic lines are usually much narrower. In this connection it is of interest to remark that Ca^+ λ 3933 is often much narrower than *He* λ 4472, even though their central intensities are nearly the same. As a matter of fact, these narrow stellar Ca^+ lines have frequently been confused with the sharp, detached (Hartmann) Ca^+ lines, but in spectral subdivisions B8 and B9 measurements of radial velocity prove them to be of ordinary stellar origin. Direct measurements also show that Mg^+ λ 4481 is, as a rule, considerably narrower than *He* λ 4472.[1]

A quantitative analysis is difficult in view of the very different central intensities. The *H* lines are strong, those of *He* are frequently faint. The average width of *He* λ 4472 in B stars is about 2.5 A, that of *Hβ* roughly 15 A. The ratio $W_H/W_{He}=6$, considerably more than would be expected from the Stark effect alone. Indeed, the maximum resolution of λ 4472 in a field of 10^5 volts per centimeter is about 20 A, while that of *Hβ* is about 40 A. The expected ratio is about 2. The remaining factor of 3 must be attributed partly to difference in abundance, and partly to the corrective term under the radical in (7).

As Miss Fairfield[2] has shown, the width of the *H* lines is closely correlated with the Mount Wilson estimate of "n" or "s." As the

[1] The spectrum of η Orionis photographed with a dispersion of three prisms reveals the presence of very narrow lines of Si^{++} and of O^+, while the *H* lines are very broad and the *He* lines rather broad (A. Pogo, *Astrophysical Journal*, **68**, 312, 1928).

[2] *Harvard College Observatory Circular*, No. 264, 1924.

latter refers chiefly to the *He* lines, there can be no doubt that the broadening mechanism is the same in these two elements.

VI. LUMINOSITY EFFECT

We have mentioned in section IV that the observed dispersion in line-widths may be partly due to differences in the Stark effect. If we consider only one line, for example, λ 4472, in a single spectral subdivision, the only quantity that can vary is F, which, as we have seen in (4), is proportional to W. For a single atom we have approximately

$$F=\frac{e}{x^2},$$

where x is the distance to the nearest charge e. Assuming that there are n free electrons per cubic centimeter, we have

$$x=c\cdot n^{-1/3},$$

and, consequently,

$$F=\text{Const. } e\cdot n^{2/3}.$$

Introducing the gas equation in the form

$$n\sim p'/T,$$

where p' is the pressure of the free electrons and T is the absolute temperature of the gas,

$$F=\text{Const. } e\cdot(p'/T)^{2/3}. \tag{12}$$

If we limit ourselves to a single spectral subdivision, we may put $T=\text{Const.}$, so that

$$F\sim(p')^{2/3}, \tag{13}$$

and, consequently,

$$W\sim(p')^{2/3}. \tag{14}$$

This relationship shows that a dispersion in W is equivalent to a dispersion in the partial electron pressures. As we shall see, this will have a marked influence upon the absolute magnitudes. The partial electron pressures may be assumed to be proportional to the total gas pressures, and the latter are, of course, related to the

[1] *Monthly Notices of the Royal Astronomical Society*, **85**, 782, 1925.

surface gravities. In fact, E. A. Milne[1] has shown that approximately

$$\frac{P_1}{P_2}=\left(\frac{g_1}{g_2}\right)^{1/2}\left(\frac{T_2}{T_1}\right)^2 . \tag{15}$$

There seems to be some slight uncertainty, however, as to the power $\frac{1}{2}$, and we shall for the present assume that, T_1 being equal to T_2,

$$p' \infty g^m . \tag{16}$$

The surface gravity is given by

$$g=\frac{\gamma \cdot \mu}{R^2} , \tag{17}$$

where γ is the constant of gravitation and μ and R are the mass and radius of the star. Consequently,

$$W=\frac{\text{Const. } \mu^{\frac{2m}{3}}}{R^{\frac{4m}{3}}} . \tag{18}$$

The radius of the star is readily expressed in terms of the absolute bolometric magnitude,[1] M:

$$log\ R=-0.2\ M-2\ log\ T+8.53 . \tag{19}$$

If we adopt Eddington's mass-luminosity relation, we can also express μ as a function of M and thus find the relationship between W and M. For simplicity we shall use Jeans's expression,[2]

$$\mu=4.85-7.5\ log\ \mu . \tag{20}$$

Substituting μ from (20) and R from (19) into (18), we get

$$log\ W=\frac{m}{5}M+\text{Const} . \tag{21}$$

Let us consider two groups of stars, the one with very narrow lines, W_1, and the second with very broad lines, W_2. Then

$$\Delta M=\frac{5}{m}\ log\ \frac{W_1}{W_2} . \tag{22}$$

[1] J. H. Jeans, *Astronomy and Cosmogony*, p. 47, 1928.

[2] *Ibid.*, p. 126.

Our actual numerical values in Table II suggest that for type B3:

$$W_1 = 1.4 \text{ A (narrow lines)},$$
$$W_2 = 2.8 \text{ A (wide lines)}.$$

The ratio W_1/W_2 is 0.5, and the corresponding value of ΔM is $+\frac{1.5}{m}$, in stellar magnitudes.

Assuming for m three different values, we find:

1. $m=\frac{1}{2}$; $\Delta M=+3.0$ mag.
2. $m=1$; $\Delta M=+1.5$
3. $m=2$; $\Delta M=+0.8$

(Stars with narrow lines should be more luminous than stars with broad lines.)

The first hypothesis agrees with Milne's theory (15). The second is that of Pannekoek. The third follows a recent suggestion of Miss Payne and F. S. Hogg,[1] who find empirically that m is greater than 1. The best representation of their observations, as a whole, is obtained with $m=2$. It must be emphasized, however, that this representation is poorest for the earliest types, and that an extrapolation of their smooth curve would give approximately $m=\frac{1}{2}$ for stars of type B.

Observational evidence shows unmistakably that A stars and B stars with narrow lines are appreciably more luminous than stars with broad lines. This effect was discovered first by Adams and Joy[2] and independently by the writer.[3] A thorough discussion by Edwards[4] fully supports these results. In the case of the *H* lines, Miss Fairfield[5] found that their widths are correlated with reduced proper motion ($h=m+5+5$ log [p.m.]), so that here, too, the requirements of the theory are fulfilled.[6]

[1] *Harvard College Observatory Circular*, No. 334, 5, 1928.

[2] *Astrophysical Journal*, **56**, 242, 1922; **57**, 294, 1923.

[3] Dissertation, University of Chicago, unpublished; *Abstracts of Theses, University of Chicago*, **2**, 1923.

[4] *Monthly Notices of the Royal Astronomical Society*, **87**, 365, 1927.

[5] *Harvard College Observatory Circular*, No. 264, 1924.

[6] This effect has also been verified by Miss A. V. Douglas in the spectra of A-stars (*Astrophysical Journal*, **64**, 262, 1926; *Journal of the Royal Astronomical Society of Canada*, **20**, 265, 1926). The relationship, discovered by Miss Payne and Miss Howe (*Harvard College Observatory Circular*, No. 287, 1925), between the number of Balmer lines visible in the spectrum and the absolute magnitude of the star, points in the same direction. Miss Payne has called my attention to this fact.

To obtain an entirely independent result I have computed the absolute magnitudes of the hotter B stars from the intensities of the detached (Hartmann) Ca^+ lines.[1] Using the relation $I_{ca}=f(\text{Dist.})$, the distance of the star is immediately found, and M is then computed by means of the apparent magnitude. It is not yet apparent whether these values are good approximations individually, but in the light of Eddington's hypothesis of interstellar matter, they should give useful results for the mean of many stars. The following values of ΔM were obtained from the Ca^+ intensities:

Spectrum	No. of Stars	ΔM	Remarks
O, Bo, B1, B2....	60	+1.2	Narrow—broad
B3.....	92	+1.9	Narrow—broad

The corresponding values of Edwards are:

Spectrum	ΔM	Remarks
Bo, B1, B2.....	+0.9	Narrow—broad
B3.....	+1.5	Narrow—broad

For B3 the mean is $\Delta M=+1.7$ mag. Adams and Joy divided the stars into two groups only, viz., "n" and "s." Consequently, they obtained a smaller value for ΔM—viz., about one-half of that given above.

The observed value of ΔM apparently agrees best with the computed value if $m=1$. However, I am inclined rather to accept Milne's result, $m=\frac{1}{2}$. It is only reasonable to assume that part of the dispersion in W is due to causes other than Stark effect. The observed difference $W_1-W_2=1.4$ A is obviously a certain multiple of σ_{obs} in (11). If the contributory dispersions are all alike, i.e., if $\sigma_{Stark}=\sigma_{rot}=\sigma_{abun}$ we have

$$\sigma_{Stark}=\frac{\sigma_{obs}}{\sqrt{3}},$$

and we see, consequently, that we should have used a smaller difference between W_2 and W_1. The corrected values would have been

[1] *Astrophysical Journal*, 67, 383, 1928.

$W_1 = 1.7$ A and $W_2 = 2.5$ A, so that the corrected ratio W_2/W_1 would have been 1.47. This would give $\Delta M = +1.7$ mag., provided $m = \frac{1}{2}$. This agrees exactly with the observations.

We see from the foregoing that the Stark effect gives a satisfactory explanation for the observed difference of luminosity between "s" and "n" stars.[1]

VII. FORBIDDEN LINES

In section II(*c*) it was stated that the presence of strong electric fields stimulates the appearance of spectral lines normally forbidden by the selection principle.[2] According to our estimate that the average intramolecular field in the reversing layers is of the order of 10^3–10^4 volts per centimeter and that fields up to 10^5 volts per centimeter are in some stars sufficiently numerous to produce appreciable absorption, it seems probable that some of the more prominent forbidden lines would be present. An inspection of the work of Takamine and Yoshida and of others shows that the near-diffuse lines of *He* make their first appearance at field intensities somewhat lower than 10^4 volts per centimeter.

Of the various elements only *He* seems to promise any results. Hydrogen shows no new lines outside the Balmer components, which are blended. All the other elements are either faint in the stars or not very susceptible to Stark effect. *He*, however, is both strong and very susceptible. A closer analysis is, therefore, justified.

In this connection the work of T. R. Merton[3] is particularly

[1] Some investigators have doubted the reality of this effect in the A stars and, in particular, Lindblad and Miss Fairfield have called attention to a systematic difference between the Mount Wilson and Harvard classifications for A stars. It is clear that the luminosity effect can either be blotted out or be spuriously introduced by an appropriate adjustment of the classification. The question reduces therefore to whether the Mount Wilson or the Harvard classification is preferable. I see no reason at present to discard the Mount Wilson system which is based upon slit-spectrograms and probably forms an excellent temperature sequence. Further investigation of this question is much needed.

[2] These forbidden lines violate the rule for the azimuthal quantum number: $\Delta k = \pm 1$. They do not involve transitions from metastable states and are therefore not restricted to low densities. While, according to Bowen, the forbidden nebular lines should be observable only in emission, there is no such restriction for lines stimulated in the electric field.

[3] *Proceedings of the Royal Society*, A, **95**, 30, 1919.

valuable. The Stark effect observed by him in a helium vacuum tube excited by means of a fairly heavy discharge from an induction coil was of the fluctuating type, and the general conditions were comparable to those in the stars. The accompanying sketch (Fig. 1),

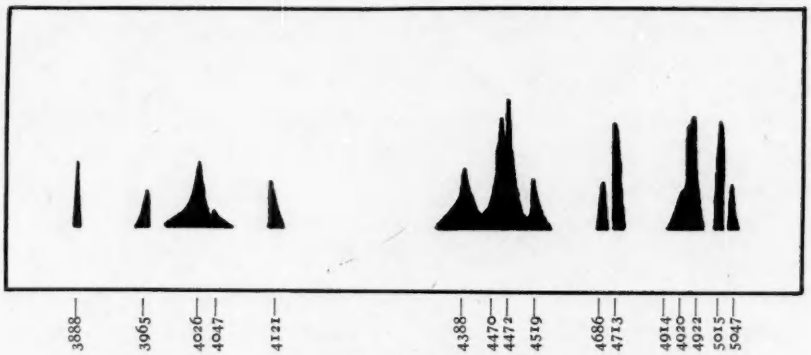

FIG. 1.—The helium spectrum in the electric field. (By R. T. Merton)

TABLE III

HELIUM LINES IN ELECTRIC FIELD

Wave-Length	Series Relationship		Remarks
4472.....	$2p^3-4d^3$	Diffuse triplet	Permitted
4470.....	2p−4f	Near-diffuse triplet	Forbidden
4922.....	2P−4D	Diffuse singlet	Permitted
4713.....	$2p^3-4s^3$	Sharp triplet	Permitted
5015.....	2S−3P	Principal singlet	Permitted
4920.....	2P−4F	Near-diffuse singlet	Forbidden
4388.....	2P−5D	Diffuse singlet	Permitted
4026.....	$2p^3-5d^3$	Diffuse triplet	Permitted
3888.....	$2s^3-3p^3$	Principal triplet	Permitted
4519.....	2p−4p	Almost-sharp triplet	Forbidden
4908.....	?	?	Forbidden
4121.....	$2p^3-5s^3$	Sharp triplet	Permitted
3965.....	2S−4P	Principal singlet	Permitted
4047.....	2p−5p	Almost-sharp triplet	Forbidden

taken from Merton's paper, represents the *He* spectrum photographed through a neutral wedge, so that the heights of the lines give an indication of their intensities. I have grouped them in Table III, roughly in order of intensity.

The strongest forbidden line is λ 4470 (given as 4470 by Merton and as 4469 by Stark), a component of the well-known line λ 4472,

which is particularly prominent in stellar spectra. The forbidden line, for which $\Delta k = 2$, was tentatively classified by Stark as belonging to the "near-diffuse" subordinate series of the helium triplet system. The next strongest forbidden line, λ 4920 ($\Delta k = 2$), lies in a region where the focus of our plates is not perfect and where we can hardly expect to see it separately from the diffuse singlet line λ 4922. There is a considerable decrease of intensity before we reach the next forbidden line, λ 4519, for which $\Delta k = 0$. We would hardly expect it to be represented in stellar spectra because it is so much fainter than λ 4470.

Hence, only one forbidden line of *He* would be expected to be present. It should lie nearly 2 A to the violet of λ 4472, and should, therefore, be separately visible in some stars. It is a fact that certain B-type stars show a faint but quite definite absorption line in that very position (λ 4470.046 A on Rowland's scale).[1] I have identified and measured this line in many stars. It appears to be strongest in or near subdivision B_3, i.e., very near to the place where the ordinary *He* lines reach their maximum intensity. I have not been able to identify it with any other element, and Mr. F. E. Baxandall informs me that he too has looked in vain for a satisfactory identification.

While no final conclusions are as yet possible, it seems reasonable to identify, tentatively, the unknown line with the near-diffuse line of *He*.

Baxandall[2] lists two other unidentified lines in the spectrum of ε Orionis (Bo), λ 4070 and λ 4655. Neither of them agrees with any of the stronger forbidden lines of *He*. I have not tried to compare them with the forbidden lines of other elements. As was stated above, the fainter forbidden lines of *He* cannot be expected to be as easily visible as λ 4470, although there is perhaps a chance that λ 4920 might be found on particularly good plates.

VIII. PRESSURE-SHIFTS

In section II(*b*) we stated that almost all spectral lines, with the exception of *H* and He^+, show more or less asymmetry in their reso-

[1] *Astrophysical Journal*, **62**, 198, 1925.

[2] *Monthly Notices of the Royal Astronomical Society*, **83**, 166, 1923.

lution into components. The mol-electric fields in the reversing layers of the stars should therefore produce spurious shifts in the positions of certain lines. It is difficult to estimate the amount of such displacements without knowing exactly the average intensity of the field and the conditions under which the various components blend together. Take, for example, *He* 4472. Its components show, on the average, a strong shift toward the red. However, at a certain stage of field intensity (something less than 10,000 volts per centimeter) the near-diffuse line at λ 4470 makes its appearance. As the field increases, this new line also broadens, chiefly toward the violet. When the two lines merge together, we should observe a single broad line slightly shifted toward the violet.

There is little information available concerning wave-lengths in stellar spectra having broad lines. Attention has been called from time to time, by various investigators, to systematic differences in radial velocities from certain individual lines, but the data are too sporadic to be of much use in this connection.

A preliminary survey of the field may be made by using the wave-lengths derived by S. Albrecht[1] from the Yerkes three-prism measures of E. B. Frost and W. S. Adams. These measures are doubtless the most accurate ones available for B-type stars. They refer to spectra with good and relatively narrow lines for which the expected Stark effect is small. Nevertheless, the lines are appreciably broadened even in these stars, and it is probable that the average intensity of the mol-electric fields in their reversing layers is of the order of 10^3 volts per centimeter. An inspection of the data collected by Stark[2] suggests that the blended line *He* 4472 for a field of 30,000 volts per centimeter is displaced toward the red by about 0.5 A. This holds true only if it is not blended with the near-diffuse line at λ 4470. In the stars under consideration there is no danger of such a blending. For a field of 10^3 volts per centimeter the red shift would amount to approximately +0.02 A, a quantity that should be easily detectable in the measurements of Frost and Adams. The line *He* 4388 will show a similar red shift. *He* 4437 should also be shifted toward the red,

[1] *Astrophysical Journal*, **67**, 305, 1928.

[2] *Handbuch der Experimental-Physik*, **21**, 399, 1927.

but by a smaller amount, and this refers also to the line *He* 4713, which, as belonging to the sharp series, is little influenced by the Stark effect.

Of the other lines measured by Frost and Adams, those due to O^+ and N^+ are not appreciably affected by the Stark effect. Neither would we expect an appreciable shift in the case of Mg^+ 4481 or of the various lines of Fe^+, Ti^+, or Si^{++}.

Albrecht's results show conclusively that the two *He* lines, λ 4388 and λ 4472, are actually displaced toward the red. The four best-determined wave-lengths of his table give the following residuals from the normal laboratory wave-lengths:

He	4388	+0.015 A
He	4472	+ .019
Mg^+	4481	− .012
Si^{++}	4552	−0.005

The displacement of the two *He* lines against the other two well-determined lines is of the order of +0.03 or +0.02 A, which agrees well with the predictions.

I have no doubt that these displacements are real, and Albrecht also considers them so. There remains, of course, a possibility that the adopted laboratory wave-lengths are in error or that there are downward convective currents of helium in the atmospheres of the stars. These possibilities should be further investigated. Unless one of them is confirmed, the observed residuals can be attributed to the Stark effect.[1]

IX. WIDTHS OF BALMER LINES

In section II(f) we stated that the Stark effect increases in size for the higher members of each series. In the case of hydrogen, equation (5) shows that

$$\Delta\lambda \propto \lambda_0^2 f \propto W \,. \qquad (23)$$

[1] *He* λ 4437 also shows a shift toward the red as predicted by the theory, but λ 4713 is shifted to the violet, while the theory demands a small red shift. I do not believe that this disagreement is fatal to the Stark effect. The weight assigned to this line by Albrecht is small (only 58 as compared with 500, for λ 4472). But even more important is the fact that λ 4713 lies outside the region of good focus on most of our plates. I have frequently had occasion to convince myself that the precision of measurement in this region is greatly reduced by the instrumental errors.

Substituting for f the values of Vasnecov (4, 10, 18, 28 for $H\alpha$, $H\beta$, $H\gamma$, and $H\delta$, respectively) and for λ_0 the wave-lengths of the Balmer lines, we should expect to find the following ratios in the line-widths:

$$W_\alpha : W_\beta : W_\gamma : W_\delta = 0.6 : 1.0 : 1.4 : 2.0 .$$

C. T. Elvey's recent measurements[1] show that in Sirius the line-widths actually increase from $H\beta$ to $H\epsilon$.[2] However, the rate of this increase[3] is much smaller than that predicted:

$$W_\beta : W_\gamma : W_\delta : W_\epsilon = 1.00 : 1.10 : 1.04 : 1.16$$

This is a serious obstacle in the way of an explanation by Stark effect. However, it is perhaps not fatal since the laboratory experiments show a similar discrepancy. Indeed, Vasnecov has tabulated Hulburt's line-widths for the H lines:

p	25	50	75	100	125 mm
$H\beta$.....	8 A	24 A	46 A	76 A	101 A
$H\gamma$.....	6	28	61	92	118
$H\delta$.....	3.5	20	60	93	105

There is surprisingly little difference between $H\beta$ and Hδ, and yet there can be no doubt as to the character of the broadening. It is possible that line-intensity, and consequently abundance-broadening, should be taken into consideration. However, surprising as this may seem, Elvey's results indicate that the central intensity increases from $H\beta$ to Hϵ.

The idea suggests itself that the insufficiency in the increase of line-width may be due to the superposition of a mol-collisional broadening.[4] This tends to give a line-width proportional to λ_0^2, and

[1] *Astrophysical Journal*, **68**, 145, 1928.

[2] Elvey expresses his line-contours by the equation $I = I_0 e^{-k(\lambda - \lambda_0)}$. Accordingly, $W = \frac{1}{k} \log_e I_0/I'$. Thus roughly $W \propto 1/k$. The value of k tabulated by Elvey shows a distinct tendency to decrease from $H\beta$ to $H\epsilon$; consequently, W increases.

[3] There is also a tendency for W to increase in the other A stars.

[4] The discrepancy between observation and theory would be removed if the coefficient of general absorption of the stellar atmosphere has a minimum value near $H\beta$, and increases toward the ultra-violet. In such a case the $H\beta$ line would originate in deeper and denser layers than the rest of the Balmer lines, and consequently the increase in width from $H\beta$ to $H\epsilon$ would be less pronounced than is indicated by formula

the result will be a partial reversal of the increase due to f in formula (23).

X. CONCLUSIONS

The various tests that we were able to apply to the hypothesis of Stark broadening in stellar spectra of early type seem, in general, to give satisfactory results.

The broadening observed in the various elements is in agreement with the requirement of the Stark effect: that the neutral atoms of low atomic weight should show wider lines than the heavier atoms.

The Stark broadening requires that stars with narrow lines of any one spectral subdivision should be more luminous than stars with broad lines. This agrees well with the observations.

The existence of lines forbidden by the selection principle of the quantum theory is suggested by the close coincidence of an unidentified line in B stars with the strongest near-diffuse line of *He*, at λ 4470.

The possibility that certain line-shifts noted by Albrecht are due to an unsymmetrical Stark broadening is in agreement with our expectations.

The only apparent diagreement is found in the amounts by which the line-widths of the *H* lines increase. The observed increase from $H\beta$ to $H\epsilon$ is smaller than that required by the computations. This may be due to superimposed effects of abundance-broadening and of mol-collisional broadening, in the sense suggested by Stark.

The luminosity effect and the possibility that forbidden lines are present, as well as the pronounced relationship with atomic number, indicate that the mechanism of broadening is dominated by the pressure. This is characteristic of the mol-electric effect but also of the mol-collisional broadening. We conclude that the broadening mechanism probably depends upon some sort of interaction between the various atoms and free electrons in the reversing layers of these early type stars.

December 12, 1928

(23). An increase of the absorption coefficient from $H\beta$ toward the red would produce a narrower $H\alpha$ line. Unsöld's contours for the solar *H* lines seem to indicate such an effect. At a point on the contour where $I = 10$ per cent absorption, the widths are approximately as follows: $W\alpha = 4.7$A; $W\beta = 5.4$A; $W\gamma = 5.6$A; $W\delta = 3.4$A. The possible variation of the absorption coefficient with λ was investigated by R. Lundblad, and by A. S. Eddington (*Der innere Aufbau der Sterne*, p. 410, 1928).

IS THERE ARGON IN THE CORONA?[1]

By H. N. RUSSELL[2] and I. S. BOWEN

ABSTRACT

Critical study of the coincidences between lines in the spectrum of the solar corona and in that of argon, which have recently been suggested by Dr. I. M. Freeman, shows: (*a*) that the number of these coincidences is little larger than might be expected from pure chance, within the very wide limits of error which have been admitted; (*b*) that the discordances for the better-measured lines greatly exceed the limits of observational error; and (*c*) that the suggested coincidences are with weak or forbidden lines, while far stronger argon lines, which should be much more persistent, are absent.

This cumulative evidence shows decisively that *the attribution of the coronal lines to argon is without foundation.*

Formulae for computing the number of chance coincidences between spectra are given and tested empirically by comparison with fictitious lines chosen at random.

Criticism of scientific work is a much less agreeable task than investigation, but it is necessary at times, for an erroneous conclusion, if not challenged, may be accepted by those without expert knowledge and may have a long life. We therefore feel under obligation to set forth the reasons which convince us that the recent attribution of most of the lines in the spectrum of the solar corona to argon[3] is without foundation.

I. THE CORONAL LINES

In the paper under consideration, Freeman lists forty-five lines as belonging to the coronal spectrum, following the list prepared in 1918 by Campbell and Moore.[4] This list, as its compilers explain, was designed to be inclusive, and contains a number of lines recorded on one occasion only, by a single observer, and regarded by him as doubtful. Davidson and Stratton,[5] after a careful discussion, reject twenty of these lines as identifiable with known radiations of the high chromosphere, and regard only sixteen as of undoubted coronal origin. St. John and his collaborators in the revision of Rowland's table[6] list seventeen lines as coronal and five more as doubtful. It

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 375.

[2] Research Associate of the Mount Wilson Observatory, Carnegie Institution of Washington.

[3] I. M. Freeman, *Astrophysical Journal*, **68**, 177, 1928.

[4] *Lick Observatory Bulletin*, **10**, 10 and 21, 1918. [5] *Memoirs, R.A.S.*, **64**, 142, 1927.

[6] *Carnegie Institution of Washington, Publication* No. 396, p. 226, 1928.

may fairly be concluded that only about half the lines on the longer list really belong to the coronal spectrum.

For many of the lines, as Freeman notes, the wave-lengths are very poor. The fainter ones have been observed only with the objective prism, which gives images of the corona itself, incapable of precise measurement. But the data for the stronger lines are much better. Davidson and Stratton during totality in 1926 obtained spectra of the corona (with iron comparison) with a large quartz slit-spectrograph, and of the flash at second contact with the same instrument. The latter shows hundreds of lines, which agree closely with the laboratory wave-lengths. For the first ten lines in each 100 A (blends and one questionable identification being excluded), the average residuals, regardless of sign, are as follows:

λ	Lines	Average Residuals
3100–3300	30	±0.044 A = ±0.43 wave-numbers
3400–3700	40	.045 = .35
3800–4100	40	±0.062 = ±0.40

The largest residual for these one hundred and ten lines is 0.19 A.

The published reproduction of the photographs shows that the coronal spectrum is of equally good quality. The seven coronal wave-lengths observed on this plate should therefore have probable errors of the order of ±0.04 A. Since the slit was set on the sun's eastern limb for half the exposure, and the western limb for the rest, the solar rotation was eliminated.

The measures in the visible spectrum are less satisfactory, except for the green line, for which the principal determinations with slit-spectrographs are:

		λ Rowland's Scale
Campbell	1898	5303.26
Campbell	1918	5302.98
Dyson	1905	5303.1
Furuhjelm	1914	5303.38
Adams, St. John, and Ware	1918	5303.24
Slipher	1918	5303.0
Davidson and Stratton	1926	5302.98

The simple mean is 5303.13 (Rowland) or 5302.95 (I.A.). The probable error of this mean found from the residuals for the seven determinations is ±0.04 A.

II. WAVE-LENGTHS

In comparing the observed coronal lines with the spectrum of argon, Freeman has admitted the very wide tolerance of accepting a coincidence if the wave-numbers agree within 5 units. The corresponding difference of wave-length ranges from 0.5 A at λ 3200 to 2 A at λ 6400. Within these limits he finds that twelve coronal lines coincide with lines of argon which Meissner[1] has classified as members of series in the arc spectrum, and six more with lines observed by him but not classified. Four other lines agree with lines of the "blue" (spark) spectrum of argon given in Kayser's *Handbuch;* and ten are identified as due to transitions between terms of Meissner's table, which do not lead to lines observed in the laboratory.

Two very serious objections may be made to these identifications: the limits of tolerance are far too wide, and the coincidences, such as they are, are with weak argon lines, while stronger lines, which are physically related to the lines in question, are absent.

"Coincidences" within such wide limits must often happen by chance. This is discussed in detail below (sec. iv); but all working spectroscopists will recognize that no decisive results can be obtained from comparisons with a rich spectrum like that of argon, unless greater accuracy is available. For the fainter coronal lines, the measures are so poor that no definite identification is at present possible. For the eight well-observed lines mentioned above, the agreement, if the lines are really due to argon, should be within 0.1 A. The facts are as shown in Table I. One of the eight lines has no argon line near it. For five more, the residuals are greater than for any one of one hundred and ten lines in the flash spectrum photographed with the same instrument two minutes earlier. One of the two remaining lines coincides with a very faint line in the spark spectrum of argon; the last agrees tolerably with two different predicted lines, neither of which has ever been observed.

[1] *Zeitschrift für Physik*, **39**, 172, 1926, and **40**, 839, 1927.

This is an extremely unfavorable showing and casts the gravest doubt upon the identification of argon in the corona.

TABLE I

WELL-OBSERVED CORONAL LINES

CORONAL LINES		ARGON LINES		Δλ	SOURCE
Intensity	λ I.A.	λ I.A.	Intensity		
20	3387.96	3388.35	5	−0.39	Meissner
5	3454.13	3454.94	5	− .81	Meissner
9	3600.97	3600.95	1	+ .02	Eder
3	3642.87	3643.09	8	− .22	Meissner
4	3800.27	3801.04	.....	− .77	Predicted
10	3986.88	.....	.....	.....	.....
8	4086.29	4085.49	.....	+ .80	Predicted
20	5302.95	5302.84	.....	+ .13	Predicted
.....	.....	5302.96	.....	−0.01	Predicted

III. INTENSITIES AND SERIES RELATIONS

The line-intensities lead to similar conclusions. For the six argon lines of type 1s−mp (or mx) which Freeman identifies with coronal lines, the intensities range from 4 to 10 on Meissner's scale. Within the same region (λ 3359–λ 3864) there are fifteen other lines, corresponding to transitions of the same type, and of intensities ranging from 10 to 50, which do not appear in the corona. The six lines of type 2p−ms (or md) which are supposed to appear in the corona have intensities between 0 and 10. Within the same region (λ 4586–λ 5597) there are twenty-three similar lines of intensity 10–20, and three lines of type 1s−mp with intensities 90 and 100, all of which fail in the corona. Finally, the six faint "coronal" lines recorded by Meissner, but not classified, have intensities from 0 to 5, while not one of ten lines of intensity 100 in the same region (λ 3950–λ 4300) appears! To cap the climax, two of these strong lines (λ 4044.419 and λ 4272.169; $1s_4-3p_3$ and $1s_4-3p_7$) are early members of series whose later and fainter members are supposed to be present (λ 3461.07 and λ 3388.35; $1s_4-4p_3$, $1s_4-5p_7$).

If the coronal lines were really due to argon, it would follow that a few faint members of certain series were excited, while far stronger lines of similar series, or even of the same series, failed to appear. Such erratic behavior, under solar conditions, is quite in-

credible. It is true that under very special conditions certain lines in a spectrum may be excited without others—for example, when the gas is stimulated to resonance by a few artificially selected wavelengths, as in Wood's experiments on mercury vapor. But, in the vicinity of the sun, excitation, whether by radiation or by collisions, must be distributed indiscriminately over a wide range of energy, and no such selective influence can occur. Even if it did, it could hardly pick out individual lines in the fashion suggested. For example, for each of the recorded lines $1s-mp$ there are two or three other lines, corresponding to transitions downward from the same excited level. However an atom of argon gets into this state, it may be expected to emit all these lines; yet the others are not recorded, though many of them are stronger than the ones which are supposed to appear.

The predicted lines remain to be discussed. Eight of them suppose transitions of the type $1s-md$ or $1s-ms'$—that is, between atomic configurations in which the excited electron is in a 4s "orbit" and a 3d—which are of the "forbidden" type. The other three transitions escape this objection; but two of them violate the inner-quantum rule, $2p_5-7s_1''$ involving a jump from 0 to 2, and $2p_5-9d_4'$ from 0 to 4 (!). Forbidden transitions may give strong lines, when the upper state involved is metastable, as in the nebulae, but such transitions from highly excited states which are not metastable have been observed only in sources where the emitting electrons are subject to strong electric fields (many thousands of volts per centimeter), and are utterly unlikely in the corona.

The only permitted line is $2p_4-9s_4$, which agrees fairly well with the green coronal line. But the series $2p_4-ms_4$ is a faint one—the observed intensities for $m=4, 5, 6, 7$ being 4, 0, 0, 0 (the last a blend with another and probably stronger line), while $m=8$ has not been observed. There can be no chance that $m=9$ gives a strong line.

Finally, four coronal lines ($\lambda\lambda$ 3237, 3505, 3601, 4130) are identified by Freeman with lines of the "blue" or spark spectrum of argon. These lines are faint, intensities 1–3, and their laboratory wavelengths are none too good. Many strong spark lines in the same region are absent.

It may finally be remarked that, if argon were present in the

corona, it should occur in greater abundance in the chromosphere, where its lines should appear, especially the very strong ones in the red and the violet. No trace of them has been found.

In view of this cumulative evidence, there appears to be no escape from the conclusion that the coronal lines are not due to argon, and that the rough coincidences of wave-length noted by Freeman are accidental.

IV. PROBABILITY OF ACCIDENTAL COINCIDENCES

A discussion of the probability of chance agreement is next in order. Freeman states[1] that he has investigated this, and finds that there is but one chance in a thousand that the relations which he has found are accidental. He gives no details. We are unable to verify his results, and our conclusions regarding probability differ radically from his.

It is simplest to go back to first principles. Suppose that a given spectrum contains M lines within an interval of X units of frequency. What is the probability that a wave-number chosen at random within the range X shall lie within x units of one or more of the spectral lines?

Consider any one line in the real spectrum. The arbitrary "line" will meet the condition if it lies within a strip of width $2x$ units, centered on the real line. The probability that this will happen is $2x/X$, and that it will not happen, $1-2x/X$. The probability that the "coincidence" will fail for all the M lines is $(1-2x/X)^M$, and the complementary probability that it will happen is

$$p=1-\left(1-\frac{2x}{X}\right)^M. \qquad (1)$$

For small values of x, this gives approximately $p=2Mx/X$; but this value is too great, since it ignores the probable overlapping of the strips centered on different spectral lines.

If now we start not with one, but with N arbitrarily chosen wave-numbers, the probability p will be the same for each. The average number C of chance coincidences within the limit x which is to be expected in a large number of trials will be given by $C=pN$.

[1] *Op. cit.*, p. 185.

If I is the average interval between the spectral lines, then $X = MI$, and

$$C = N\left\{1 - \left(1 - \frac{2x}{IM}\right)^M\right\} = N\left(1 - e^{-\frac{2x}{I}}\right). \qquad (2)$$

The latter value, which is the limit of the preceding expression when M is great, is a sufficient approximation in practice.

In the case of the coronal lines, Dr. Freeman finds the average interval between each identified line and the next adjacent argon line on each side to be 30 wave-numbers. This is then the average value of I for the part of the argon spectrum under consideration. With $N = 45$, $x = 5$, equation (2) gives $C = 12.9$. Thirteen coincidences of coronal and argon lines, within the assigned limits of error, might be expected as a result of chance.

In individual cases deviations from the mean value are to be expected, on account of the error of random sampling. If p is the probability that a given event will happen in one trial, the probability q that it will occur r times out of N trials is given by the equation[1]

$$q = \frac{1}{\sqrt{2\pi p(1-p)N}}\, e^{-\frac{(r-pN)^2}{2Np(1-p)}}. \qquad (3)$$

That is, the individual values found in a large number of trials will be distributed about the mean value Np with a standard deviation given by the equation

$$\sigma^2 = Np(1-p).$$

In the present case, $N = 45$, $p = 0.29$, and $\sigma = \pm 3.0$. The corresponding probable error is 2.2, so that we may state the chance expectation more precisely as 12.9 ± 2.2 coincidences. The observed number (for lines of the arc spectrum) is 18. The difference is hardly significant. The average residual in the case of accidental coincidences may readily be calculated. By (1) and (2) the probability that an arbitrarily chosen wave-number will be within the limits x and $x + dx$ from the nearest spectral line is

$$dp = \frac{2}{I}\, e^{-\frac{2x}{I}} dx.$$

[1] Burnside, *Theory of Probability*, p. 44, 1928.

The mean value of the distance, for all cases in which it is less than x, will be

$$\bar{x}=\frac{1}{p}\int_0^x x\,dp=\frac{I}{2}-\frac{xe^{-\frac{2x}{I}}}{1-e^{-\frac{2x}{I}}}=\frac{x}{2}\left(1-\frac{x}{3I}+\frac{x^3}{45I^3}\cdots\right). \qquad (4)$$

Substituting $I=30$, $x=5$, we find $\bar{x}=2.36$. The observed residual, regardless of sign, is ± 2.1—a good agreement.

The approximately equal numbers of positive and negative residuals, which Freeman takes as evidence that they represent the accidental errors of observation, are of course equally well explained by chance.

It appears, therefore, that the coincidences between the coronal lines and those of the arc spectrum of argon are of substantially the character which should be found if they are of accidental origin. This may be tested further by considering the numbers of coincidences within closer limits. For the eighteen arc lines in Freeman's Table II, the observed distribution of the residuals, without regard to sign, and that to be expected by chance, are as follows:

	Observed	Computed
0	4	1.3
1	4	3.0
2	3	2.7
3	2	2.5
4	3	2.2
5	2	1.2
Total	18	12.9

In computing the chance distribution from (2), it is assumed that all residuals less than 0.5 are recorded as "0," those between 0.5 and 1.5 as "1," and so on, except that "5" means residuals between 4.5 and 5.0.

On the face of this evidence, there might be some real coincidences among the lines with residuals of 0 or 1. These lines are as follows:

	Residual 0		
	Corona	Argon	Notes
3461	Faint	(10)	*Ti*+
5073	Faint	(8)	*Ti*+
5118	Faint	(6)	.....
5597.5	Faint	(10)	.....

		Residual ±1	
3328.1	Fairly strong	(1)	*Ti*+
3455	Strong	(5)	Coronal line
4241	Faint	(0)	.....
5433.4	Rather faint	(0)	.....

The only strong coronal line among the eight shows an altogether inadmissible residual, according to the measures of Davidson and Stratton (Table I). Three of the others are identified by these observers as chromospheric lines of *Ti*+. The remaining four are all faint in the corona, and our physical arguments apply to them with full force.

The four lines which are identified with argon spark lines must be considered separately since this is a quite independent spectrum. They lie between wave-numbers 31,000 and 24,000. Within this region there are one hundred and twenty-three arc lines in Meissner's lists and two hundred and forty-three spark lines in Kayser's *Handbuch*. The number of chance coincidences should therefore be greater than for the arc. Within this region Freeman gives six coincidences with arc lines. There are ten coincidences with spark lines, within his limits, but when there is also agreement with an actual or predicted arc line, he prefers the latter. Here again the agreement is evidently due to chance.

In view of the importance of the conclusion that the observed agreements are accidental, we have tested the matter further in a way which takes into account the unequal density of distribution of the argon lines in different regions of the spectrum. The whole observed range has been divided into intervals of 1000 wave-numbers and formula (2) applied to each interval successively (the factor $1-2x/X$ having now the fixed value 0.99). The results are given in Table II. The sum of the resulting values of C gives the whole number of coincidences to be anticipated as 13.6, slightly greater than that of the simpler approximation.

It should be noticed that the numbers of observed coincidences rise and fall in striking parallelism with the predicted values—which is further evidence of their accidental origin.

To find the probability of coincidences between the coronal and predicted argon lines, the whole number of differences between

Meissner's terms which fall between given limits was calculated. By subtracting the number of observed series lines between these limits, the number of possible predicted lines between these limits was found. By proceeding as in Table I and considering those coronal

TABLE II

CALCULATED NUMBER OF COINCIDENCES BETWEEN CORONAL AND ARGON LINES ALLOWING TOLERANCE OF 5 FREQUENCY UNITS

Frequency Range	*M* Number Argon Lines	*p* Probability of Coincidence per Line	*N* Number Coronal Lines	*C* Probable Number of Coincidences	Observed
15,000					
16,000	32	0.28	1	0.28	0
17,000	57	.44	0	0.00	0
18,000	67	.49	2	0.98	1
19,000	94	.61	5	3.05	2
20,000	92	.60	2	1.20	2
21,000	77	.54	1	0.54	1
22,000	44	.36	4	1.44	2
23,000	32	.28	3	0.84	0
24,000	31	.27	4	1.08	3
25,000	7	.07	2	0.14	0
26,000	8	.08	3	0.24	1
27,000	5	.05	1	0.05	0
28,000	18	.17	6	1.02	1
29,000	20	.18	4	0.72	2
30,000	26	.23	2	0.46	2
31,000	39	.32	3	0.96	1
32,000	34	0.28	2	0.56	0
Total..	.....	.....	.....	13.56	18

lines which had not already been identified with arc lines of argon, the whole number of coincidences which might be expected by accident was found to be 11.17. The observed number is 10. Here less is found than should usually appear by chance.

V. FURTHER TESTS OF PROBABILITY

To test still further the validity of equation (2) we have taken other sets of numbers by modifying the coronal wave-numbers in various ways—first, by adding or subtracting arbitrary amounts, and then by substituting, for the last two or three figures of the coronal data, figures obtained by putting a pencil down blindly on

a table of logarithms. The latter test was repeated twenty-five times to secure statistically significant results. The results are given in Table III.

TABLE III

COINCIDENCES OF ARGON LINES WITH 45 ARBITRARILY SELECTED WAVE-NUMBERS

	Number of Coincidences	Average Deviation		Number	Average Deviation	Number	Average Deviation
Corona	18	2.2	Corona with last three figures at random	10	1.9	15	2.3
Corona −100	17	2.1		11	2.4	15	1.9
Corona +100	17	2.4		13	2.0	10	2.4
Corona −10*N*	14	1.7		16	3.0	14	1.8
Corona +10*N*	13	2.2		16	2.4	13	2.2
Last two figures at random	13	1.8		15	2.1	14	1.9
	19	2.3		14	3.3	15	2.9
	9	2.9		12	2.1	16	2.9
	15	2.5		14	2.1	10	2.5
	17	2.1		12	2.4	10	2.3

N denotes the serial number of the line in the list.

For the shifts of one hundred wave-numbers, which destroy all the physical significance of the data, the number of coincidences is only one less than for the coronal lines themselves.

TABLE IV

NUMBER OF COINCIDENCES WITH RANDOM "LINES" IN 25 TRIALS

No. of Coincidences	Calculated Expectation	Observed in 25 Trials	No. of Coincidences	Calculated Expectation	Observed in 25 Trials
7	0.3	0	15	2.9	5
8	0.7	0	16	2.3	3
9	1.1	1	17	1.7	1
10	1.7	4	18	1.1	0
11	2.3	1	19	0.7	1
12	2.9	2	20	0.3	0
13	3.2	3	21	0.2	0
14	3.2	4	22	0.1	0

For the twenty-five cases in which the last figures are purely arbitrary, the mean number of coincidences is 13.52, as against 13.56 calculated in Table II. The average deviation of these arbitrary numbers from the argon lines, for the whole series, is ± 2.34, while equation (4) gives 2.36, if applied to the data as a whole with

$2x/I = 1/3$, and 2.27, if applied to successive intervals, as in Table II.

The distribution of the number of coincidences actually found in the various trials compares with that predicted by formula (3) as indicated in Table IV.

The agreement is good. The standard deviation from the mean, 13.5, is ± 2.5—less than the predicted value 3.0; but the probable error of determination of the standard deviation from n observations is $0.675\sigma/\sqrt{2n}$, or ± 0.40 in this case, so that the difference is not significant. This empirical test therefore confirms the theoretical formulae at all points.

VI. CONCLUSION

The result of the present investigation is definite. The number of coincidences between the coronal lines and those of argon is very little greater than would usually occur by chance, within the very wide limits of error which have been adopted, and is no greater than has been found, in a moderate number of trials, with "lines" chosen quite at random. So far as considerations of probability go, a small fraction of the apparent coincidences might be real; but the total absence of all the stronger and more characteristic lines of argon from the corona settles the question in the negative. For the well-measured coronal lines, moreover, the discordance of wave-lengths alone puts the identifications out of court. When critically considered, therefore, there is no spectroscopic evidence of the presence of argon in the corona, nor, for that matter, elsewhere in the sun.

The coronal spectrum remains a mystery. A careful search by one of us (I. S. B.), to be reported elsewhere, has revealed no coincidences with forbidden lines of other elements—a fact which is probably explained by the intensity of radiative excitation as suggested by Eddington,[1] and we have at present no solution of the riddle to offer.

In conclusion we desire to impress upon observers of coming eclipses the great importance of observations for the definite dis-

[1] *Monthly Notices, R.A.S.*, **88**, 134, 1927.

crimination between coronal and chromospheric lines, and of precise determinations of wave-length for the latter with slit-spectrographs. Without such data a positive identification of most of the coronal lines is hardly possible.

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A SPECTROSCOPIC DETERMINATION OF THE PRESSURE IN THE CALCIUM CHROMOSPHERE

AN EXPERIMENTAL TEST OF MILNE'S THEORY[1]

By ALBRECHT UNSÖLD[2]

ABSTRACT

1 and 2. *Application of the theory of radiative equilibrium* to the emission and absorption spectrum of the calcium chromosphere.

3. *The coefficient of scattering.*—The dependence on wave-length is largely the result of Doppler effect which is produced by a kind of turbulent motion in the chromosphere.

4. *Observations of the H and K emission and absorption lines* with the 150-foot tower telescope.

5. *Comparison of the observations with the theory* developed in sections 1 and 2. By determining the numerical values involved in this theory the $Ca+$ pressures in the chromosphere can be measured.

6. *Comparison with Milne's theory.*—Deviations from thermodynamic equilibrium in the chromosphere.

7. *Dynamics* and *stability* of the chromosphere.

8. Theoretical remarks concerning the *distribution of pressure* in the chromosphere and the theory of Milne.

INTRODUCTION

In a previous paper the author[3] has given an explanation of the self-reversals[4] observed in the emission spectrum of the limb of the sun. It appeared that the structure of these emission lines is not determined, as in the case of absorption lines, solely by the total optical depth of the solar atmosphere for the corresponding wave-length, but depends on the pressure of the chromosphere itself. In the present paper the theory will first be extended for the case of the Ca^+ resonance lines, H and K. It will then appear that the spectral distribution of the coefficient of scattering of the upper layers of the chromosphere does not depend primarily on resonance-scattering as in the reversing layer, but is essentially the result of Doppler

[1] *Contributions from The Mount Wilson Observatory, Carnegie Institution of Washington*, No. 377.

[2] Fellow of the International Education Board.

[3] *Zeitschrift für Physik*, **46**, 782, 1928.

[4] These were previously observed by G. E. Hale and F. Ellerman, *Yerkes Observatory Publications*, **3**, Part 1, 1903, and by H. Deslandres, *Annales de l'Observatoire de Paris-Meudon*, **4**, Part 1, 1910.

effect of the Ca^+ atoms. The observations indicate velocities of about 15 km/sec.

With the aid of this coefficient of scattering the pressure and even the variation of pressure with height in the chromosphere may be deduced, as regards order of magnitude, from the observations. Pressures of $\sim 10^{-15}$ atm. are obtained.

If the chromosphere is really supported by radiation pressure, the radiation which is removed by the chromospheric absorption lines, H_3 and K_3, must exactly suffice to support the mass of Ca^+ corresponding to the foregoing pressure. We shall see that this fundamental requirement of Milne's theory is actually fulfilled. Finally, we shall add some further considerations about the dynamics of the chromosphere. In particular, we shall show that theoretically there may be a continuous transition between the ordinary chromosphere and the prominences.

1. APPLICATION OF THE THEORY OF RADIATIVE EQUILIBRIUM TO THE EMISSION SPECTRUM OF THE SOLAR LIMB

Before we can calculate the distribution of intensity of the emission spectrum we must find the emissivity, J, of the solar atmosphere as a function of the optical depth. For this purpose the curvature of the layers may be neglected, since the height H of the chromosphere is small in comparison with the radius R of the sun.

The solar chromosphere does not receive its radiation directly from the photosphere, but through the reversing layer, which produces the broad H_1 and K_1 lines. In these deep layers the pressures are still comparatively high (10^{-4} to 10^{-7} atm.). Therefore, in addition to pure resonance-scattering, collisions between Ca^+ atoms and other particles will play an important part. Consequently, as the author has shown elsewhere,[1] there remains in the middle of these lines a residual intensity which must be equal for H and K. We take its numerical value from the observations as $\frac{I}{r} \sim \frac{I}{10}$ of the intensity of the undisturbed continuous spectrum of the center of the sun. The dependence of this residual intensity on direction is given, theoreti-

[1] *Probleme der modernen Physik: Festschrift zum 60. Geburtstag von A. Sommerfeld*, Leipzig, 1928.

cally, as well as by observations, for $r \gg 1$, by $1+\frac{3}{2}\cos\vartheta$, where ϑ is the angle between the direction of the radiation and the normal to the solar surface. Thus, at the lower limit of the chromosphere, the outward radiation is

$$I_0=\frac{2}{5}\cdot\frac{1}{r}I_1\left(1+\frac{3}{2}\cos\vartheta\right). \tag{1}$$

In the outer chromosphere we can surely neglect the effect of collisions and so concern ourselves with pure scattering. We denote by $\bar{s}$ the mean value per centimeter of the coefficient of scattering for the whole chromosphere. The mean value for all layers whose distance from the outer boundary of the chromosphere is $\leqq y$ is denoted by s. We have designated the total height of the calcium chromosphere by H. If we neglect the reaction of the chromosphere on the reversing layer, then, according to Milne,[1] the emissivity of the chromosphere is

$$J=\frac{2}{5}\cdot\frac{1}{r}I_1\frac{1+\frac{3}{2}sh}{1+\bar{s}H}. \tag{2}$$

Evidently $\bar{s}=s(H)$; the dependence on the wave-length is the same for both $\bar{s}$ and s. If we take into account the backward radiation of the chromosphere and solve the Schwarzschild differential equations for an atmosphere which, in its lower part (reversing layer), exerts both scattering and absorption, but in its upper part (chromosphere), scattering only,[2] we obtain, instead of (2), the equation

$$J=\frac{2}{5}\cdot\frac{1}{r}I_1\frac{1+\frac{3}{2}sh}{1+\frac{2\omega}{1+\omega}\bar{s}H}. \tag{2'}$$

The coefficient ω is determined[3] from $\omega^2/1-\omega^2$, the ratio of the coefficient of absorption κ to coefficient of scattering σ for the reversing

[1] *Philosophical Transactions of the Royal Society of London*, A, **223**, 201, 1922; A. Unsöld, *Zeitschrift für Physik*, **46**, 765, 1928, esp. pp. 767 ff.

[2] Cf. *Mt. Wilson Contr.*, No. 378; *Astrophysical Journal*, **69**, 278, 1929.

[3] We adopt here the designation of the above-mentioned paper.

layer. As was shown in a former paper already cited, $\frac{\kappa}{\sigma}=0.044$ for the H and K lines, and, therefore, $\frac{2\omega}{1+\omega}\approx\frac{1}{3}$.

The observable brightness I of the spectrum of the limb is now determined exactly in the manner previously described[1] by integrating along the line of vision over the emissivity multiplied by the corresponding absorption factor. If we designate by y the distance of the line of vision from the outer boundary of the chromosphere, we obtain by the method already described

$$I=\frac{\frac{2}{5}\cdot\frac{I_1}{r}}{1+\frac{2\omega}{1+\omega}\bar{s}H}\left\{\left(1-\frac{3}{2Rs}+3\sqrt{\frac{y}{2R}}\right)-e^{-2s\sqrt{2Ry}}\left(1-\frac{3}{2Rs}-3\sqrt{\frac{y}{2R}}\right)\right\}. \quad (3)$$

2. THE ABSORPTION SPECTRUM OF THE CHROMOSPHERE

To the Ca^+ emissions observed at the solar limb there correspond in the absorption spectrum of the solar disk two narrow depressions in the middle of the H and K lines, the so-called H_3 and K_3 lines, whose distribution of intensity F_a is determined by the total optical depth $\bar{s}H$ of the chromosphere alone. If we confine ourselves for the sake of simplicity to the integrated solar radiation[2] and make use of the circumstance that for all wave-lengths $\frac{2\omega}{1+\omega}\bar{s}H<1$, the formulae derived in the former paper[3] give

$$F_a=\frac{F_0}{1+\frac{2\omega}{1+\omega}\bar{s}H}, \quad (4)$$

where F_0 is the intensity in the middle of the H and K lines for the integrated solar spectrum. This formula allows us to determine from the observed structure of the H_3 and K_3 lines the total optical thickness of the chromosphere as a function of wave-length. The rough

[1] A. Unsöld, *op. cit.*, p. 782.

[2] For pure scattering it is known that the dependence of the form of lines on direction is very small.

[3] A. Unsöld, *op. cit.*; esp. p. 768.

picture thus obtained can afterward be refined with the help of equation (3) by observations of the corresponding emission spectrum. As we shall see, these confirm, first, the belief that the H_3 and K_3 lines originate in the outer chromosphere, and allow, moreover, a rough estimation of the pressure gradient in the chromosphere. Before we start with the description and evaluation of the observations, however, it is desirable to make some remarks concerning the coefficient of scattering s or $\bar{s}$.

3. THE COEFFICIENT OF SCATTERING

In previous papers concerning the structure of solar lines we have shown that the coefficient of scattering in the neighborhood of a resonance line is determined by radiative damping. The broadening of lines by Doppler effect could be neglected. The broadening corresponding to the thermal motion of atoms in the solar atmosphere is of the order of 1/100 A, which can be neglected, and there are no other motions which could have any considerable effect on the broader lines. There is a temptation to apply also to the chromosphere the well-known formula[1]

$$s=\frac{2\pi e^4 \lambda_0^2 Nf}{3\, m^2 c^4 \Delta\lambda^2}. \tag{5}$$

Great difficulties are met in doing this, however. The middle of the H_3 and K_3 absorptions and emissions should show practically complete darkness, as the factor $\bar{s}H$ in the denominator of formulae (3) and (4) becomes enormously great. As we shall see, this is not in accordance with the observations; moreover, such an atmosphere could not be supported by the light-pressure. An explanation of the residual intensity in the middle of the lines by the assumption of collisions, as in the case of the reversing layer, would scarcely be possible on account of the very low pressures of the chromosphere.

[1] Cf., for instance, *ibid.*, p. 768. The designations are:
e,m = electronic charge and mass
c = Velocity of light
λ_0 = Wave-length of the spectral line
$\Delta\lambda$ = Distance of the place observed (λ) from the spectral line (λ_0)
N = Number of atoms per cubic centimeter
f = Oscillatory power ($f=\frac{1}{2}$ and $\frac{2}{3}$ for H and K)

A possibility of avoiding these difficulties and, as far as I see, the only one, is indicated by a consideration of the geometrical structure of the chromosphere. As the spectroheliograms show, the outer chromosphere is not a quiet gaseous layer, as was assumed in deriving (5), but resembles more a burning prairie. These observations suggest the assumption that the whole chromosphere is in a kind of turbulent motion with an average velocity v. For the sake of simplicity we consider the distribution of velocities as Maxwellian, although naturally this is quite unimportant. As an approximate calculation easily shows, it is sufficient for what follows to compute the coefficient of scattering for the case of Doppler width $\gg$ damping width. Because of the Maxwellian distribution of velocity, its dependence on the wave-length is given by

$$s = C \cdot e^{-\left(\frac{\Delta\lambda}{\Delta\lambda_0}\right)^2}, \tag{6}$$

where $\Delta\lambda_0 = \lambda_0 \frac{v}{c}$ is the Doppler shift corresponding to the most probable velocity. The constant C is most easily determined by means of Tolman's relation between the "total absorption" and the Einstein transition probability B_{12}. We thus have

$$\frac{1}{N}\int s\,d\nu = \frac{h\nu}{c} B_{12}, \tag{7}$$

the integral being extended over the H and K lines. But, on the other hand, B_{12} is determined by the sum rule of Kuhn, Thomas, and Reiche,[1] which gives

$$B_{12} = \frac{\pi e^2}{mh\nu}. \tag{8}$$

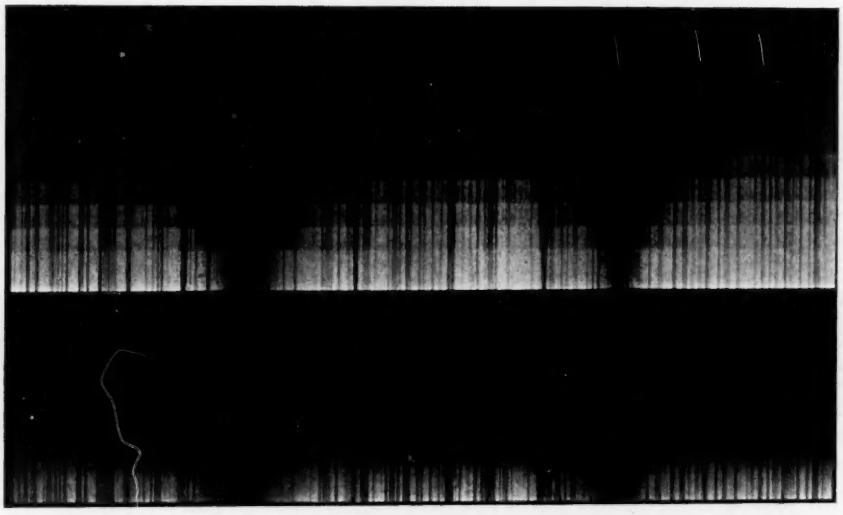
We thus obtain directly:[2]

$$s = \frac{\sqrt{\pi}\, e^2 \lambda_0^2 N f}{mc^2 \Delta\lambda_0} e^{-\left(\frac{\Delta\lambda}{\Delta\lambda_0}\right)^2}. \tag{9}$$

[1] *Zeitschrift für Physik*, **34**, 510, 1925.

[2] First derived in a somewhat different way by W. Voigt, *Sitzungsberichte der Akademie der Wissenschaften zu München*, p. 603, 1912; esp. p. 617. The earlier literature on the subject is given there.

PLATE IX



CHROMOSPHERIC SPECTRUM WITH INTENSITY MARKS

UNIV. OF MICH.

Having prepared the necessary foundation, we may now begin in a following section the description and evaluation of the observations.

4. OBSERVATIONS OF THE EMISSION AND ABSORPTION SPECTRA OF THE CALCIUM CHROMOSPHERE

By means of the 150-foot tower telescope, a solar image 42.6 cm in diameter was projected on the slit of the 75-foot grating spectrograph. In order to obtain the emission spectrum of the chromosphere, the solar limb was so placed that it formed an angle of about 20° with the slit. The spectrum was photographed in the first order (1 mm = 0.7 A). The photometric scale was obtained in the manner previously described.[1] By means of a rotating sector with stair-shaped sections, spectra of the center of the solar disk having graded intensities were photographed on the same plate. As very great intervals of intensity were to be covered, Eastman 40 plates were used. The photographs of the narrow H_3 and K_3 absorptions in the spectrum of the solar disk were taken with the third order of the same spectrograph. For the photometric measures the thermoelectric registering photometer of the Mount Wilson Observatory was used. A superficial survey of the material showed at once that the emission and especially the absorption spectrum of the chromosphere is subject to considerable variation in different parts of the sun. The numerical values can therefore claim no very great accuracy. They are, however, as may easily be seen, fully sufficient for our purposes.

Plate IX shows one of the chromospheric spectra thus obtained. Figures 1–4 give photometric traces of the H and K lines (besides H, the hydrogen line H_ϵ appears in emission) at altitudes of 2500 and 5600 km above the photosphere. On Plate IX, 1 mm corresponds to about 1000 km.

5. NUMERICAL EVALUATION OF THE OBSERVATIONS AND COMPARISON WITH THE THEORY

We come now to the most important part of the present investigation, the measurement of the pressure in the chromosphere. As already mentioned, the attempt to account for the observations was

[1] A. Unsöld, *op. cit.*, p. 769.

first made under the assumption of a quiescent chromosphere. The coefficient of scattering would then be determined by equation (5) of section 3. According to this formula, however, as can be seen by means of (3) and (4), the middle of the absorption and emission

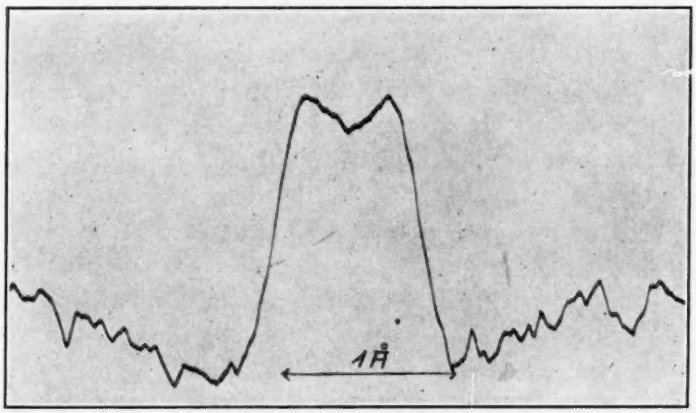


FIG. 1.—K emission 2500 km above the photosphere

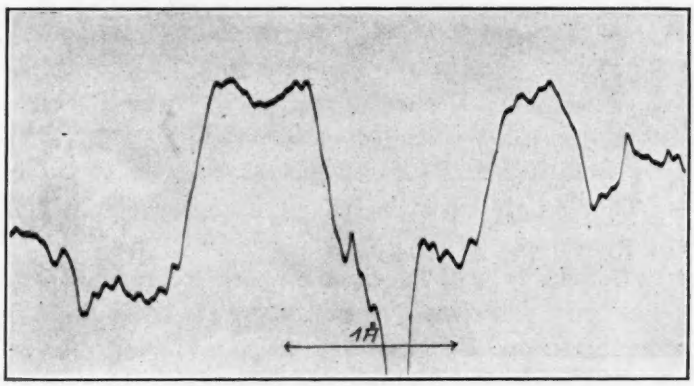


FIG. 2.—H and H_ϵ emission 2500 km above the photosphere

lines ought to be perfectly dark. Furthermore, since in (5) the distance from the center of the line, $\Delta\lambda$, occurs only in the combination $f/(\Delta\lambda)^2$, the widths of the H and K emission (and absorption) lines should have the ratio of $1:\sqrt{2}$. None of these predictions is fulfilled, for the middle portions of the lines show considerable central intensity, and the ratio of the widths of the emission lines is

about 1:1.1 instead of 1:1.4. Equation (5) cannot therefore be applied to the chromosphere. As was shown in section 3, the only way out of this dilemma is the assumption that the chromosphere is permanently in a kind of turbulent motion. This hypothesis gives

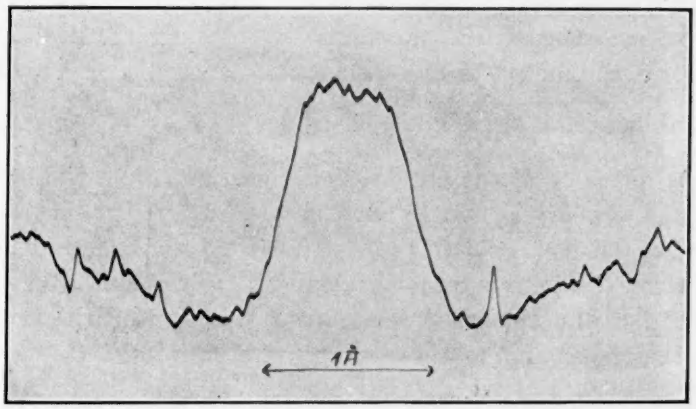


FIG. 3.—K emission 5600 km above the photosphere

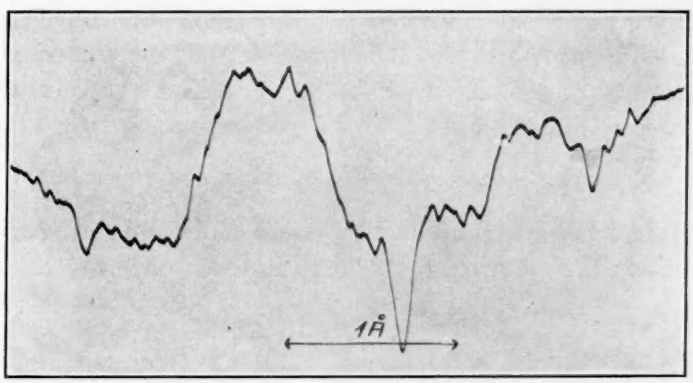


FIG. 4.—H and H_ϵ emission 5600 km above the photosphere

formula (9) for the coefficient of scattering. We will now show that this assumption gives a satisfactory representation of the observations and furnishes reasonable values for the pressures in the chromosphere. Here and in what follows we are, naturally, dealing always with the Ca^+ partial pressure.

Numerical values must now be introduced into equations (9),

(4), and (3). The solar radius is $\sim 7\times 10^{10}$ cm. The height of the Ca^+ chromosphere, according to S. A. Mitchell,[1] is 14,000 km $= 14\times 10^8$ cm. Since the photometric traces refer to heights of 2.5×10^8 and 5.6×10^8 cm, the values for y, the distance of the line of vision from the outer boundary of the chromosphere, are 11.5×10^8 and 8.4×10^8 cm. For a first approximation, consider the pressure in the upper chromosphere as constant, an assumption that will be dropped later, and write in (3), for trial, $s=\frac{1}{3}\bar{s}$.

The depth of the absorption lines, measured from the bottom F_0 of the ordinary H and K lines, is $\sim$ 25–30 per cent;[2] hence, for $\Delta\lambda = 0$ we put in (4) $F_a \approx 0.75\, F_0$. Thus we can calculate first $\bar{s}H$, then $\bar{s}$, and finally, by means of (9), the value of $N/\Delta\lambda_0$ where N is the number of atoms per cubic centimeter and $\Delta\lambda_0$ is the Doppler width corresponding to the average velocity of the Ca^+ atoms. Either the width of the absorption lines or the value of $F_a : F_0$ for any $\Delta\lambda \neq 0$ makes it possible by means of equation (9) to determine $\Delta\lambda_0$ itself and therefore also the value of N. At the same time, the intensity of the emission lines is fixed throughout the chromosphere by the values of N and $\Delta\lambda$. By successive trials the numerical values were improved somewhat further, and then the calculations were carried through for $\Delta\lambda_0 = 0.2$ Å, corresponding to a velocity of $\frac{0.2}{3950}\times 3.10^5$ km/sec. $= 15$ km/sec., and for $N = 5700$ atoms per cubic centimeter (the average over the whole chromosphere). For $\bar{s}$ we then obtain the following formulae in which $\Delta\lambda$ is measured in Å,

$$\left.\begin{aligned} \mathrm{K_3\ line:}\quad & \bar{s}_K = 15\times 10^{-10} e^{-\left(\frac{\Delta\lambda}{0.2}\right)^2}, \\ \mathrm{H_3\ line:}\quad & \bar{s}_H = 7.5\times 10^{-10} e^{-\left(\frac{\Delta\lambda}{0.2}\right)^2}. \end{aligned}\right\} \qquad (10)$$

Figure 5 gives a graphical representation of these formulae. Here and in what follows the full curve refers to the K line; the dotted curve, to H. All the curves are drawn for only one side of the lines. By

[1] *Astrophysical Journal*, **38**, 407, 1913.

[2] The difference of this value for H_3 and K_3 can scarcely be measured.

equation (4) the whole distribution of intensity of the absorption lines can now be calculated. Figure 6 shows the curves thus obtained.

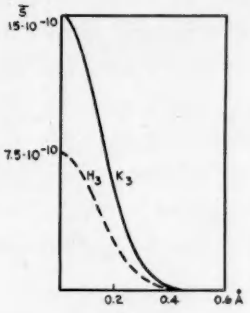


FIG. 5

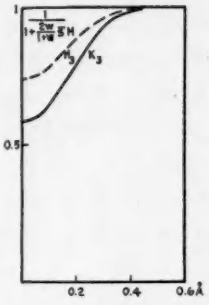


FIG. 6

FIG. 5.—Scattering coefficient for H_3 and K_3
FIG. 6.—Contours of H_3 and K_3 absorption lines

Let us now consider the emission lines. In Figure 7, again under the assumption $s=\frac{1}{3}\bar{s}$, we have plotted by means of equation (3) the intensity I of the emission, in units of $\frac{2}{5}\frac{I_1}{r}$, as function of $\bar{s}$

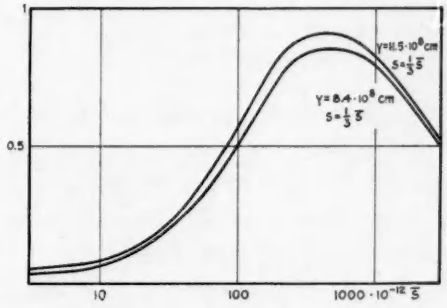


FIG. 7

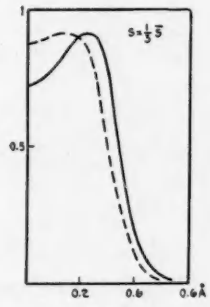


FIG. 8

FIG. 7.—Intensity of emission as function of $\bar{s}$
FIG. 8.—Emission lines for $y=11.5\times10^8$ cm

(abcissae in logarithmic scale) for the two values of y corresponding to our observations. With the help of the curves in Figure 5 we can now draw the profiles of the corresponding emission lines. Figures 8 and 9 show the curves obtained in this way. The general agreement with observation is fairly satisfactory. The ratio of the line

widths is really 1:1.1, as the observations require.[1] In the same way the characteristic dip in the center of the lines results directly from our formulae. The agreement of the calculated absorption lines with observation is also satisfactory. Only in one respect is there still a discrepancy between theory and observation. The intensity in the middle of the emission line decreases much more rapidly with increasing altitude above the photosphere, that is, with values of $H-y$, than our calculations require. Furthermore, the intensity in the middle of the emission lines ($I_{\Delta\lambda=0}$) is smaller in comparison with the continuous spectrum of the center of the sun, I_1, than our pres-

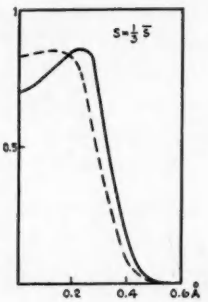


FIG. 9

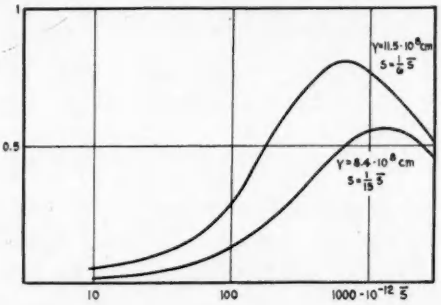


FIG. 10

FIG. 9.—Emission lines for $y=8.4\times10^8$ cm
FIG. 10.—Intensity of emission as function of $\bar{s}$

ent theory demands. As seen from Figure 8, this gives, for $y=11.5\times10^8$ cm, the value $I_{\Delta\lambda=0}\sim0.8\times\frac{2}{5}\frac{I_1}{r}$, or, with $r\approx11$ (Schwarzschild), $I_{\Delta\lambda=0}\approx0.03\ I_1$. The observations give about $I_{\Delta\lambda=0}\approx0.01\ I_1$, but this numerical value has a considerable uncertainty.

These differences between the observations and our calculations indicate clearly that the assumption of a constant value for s over the whole outer chromosphere is only a very rough approximation. We must therefore use in equation (3) different numerical values of s for different heights. The dependence on the wave-length, in

[1] Also the ratio of intensities of the *Ca*+ lines, K:H=1:0.8, measured by C. R. Davidson, *Monthly Notices, R.A.S.*, **88**, 30, 1927, agrees well with our results. The measures by C. R. Davidson, M. Minnaert, L. S. Ornstein, and F. J. M. Stratton, *ibid.*, p. 536, which give an intensity ratio of 1:0.57, refer apparently to a prominence, to which our calculations naturally cannot be applied.

other words, the average velocity of the Ca^+ atoms, we shall assume to be constant. The value of $\bar{s}$ naturally cannot be changed, as it is fixed by the absorption lines. After some trials it was seen that the observations can be represented best by putting $s=\frac{1}{6}\bar{s}$ for $y<11.5\times 10^8$ cm and $s=1/15\ \bar{s}$ for $y<8.4\times 10^8$ cm. Figure 10 shows for these cases I (in the former unit) as a function of $\bar{s}$. It will be noted that the maximum of the emission, especially for the higher layers, has shifted toward increasing $\bar{s}$, that is, toward the center of the line, and becomes lower at the same time.

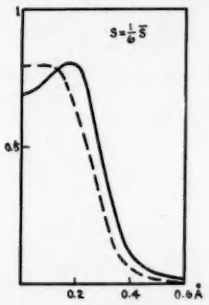


FIG. 11

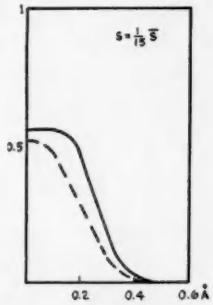


FIG. 12

FIG. 11 —Emission lines for $y=11.5\times 10^8$ cm
FIG. 12.—Emission lines for $y=8.4\times 10^8$ cm

The distributions of intensity for the H and K lines calculated for $y=11.5\times 10^8$ cm ($s=1/6\ \bar{s}$) and for $y=8.4\times 10^8$ cm ($s=1/15\ \bar{s}$) are shown in Figures 11 and 12.

The agreement with the observations is now a great deal better, but the calculated total intensity is still too great, and the observed decrease of $I_{\Delta\lambda=0}$ with elevation above the photosphere which is expressed by

$$\frac{I_{y=11.5\times 10^8}}{I_{y=8.4\times 10^8}}\sim 3$$

is also greater than our calculations indicate. A much greater decrease of the pressures toward higher layers cannot well be assumed, because a comparison of curves 7 and 10 shows that the characteristic minimum of intensity in the middle of the emission lines would then disappear. This remaining disagreement cannot, however, be

considered a serious objection to the theory, since the assumption that $\Delta\lambda_0$ is independent of y and that s can be considered as constant for $y >$ some given value furnishes at best only a rough picture of the reality.[1]

All things considered, the uncertainty of the pressures derived in the following paragraph should not be much greater than a factor of 10.

Assuming a temperature of $T=4800°\ K$, the exact numerical value of which is not very essential, we obtain for the average pressure of the chromosphere

$$p=NkT=4\times10^{-9}\ \text{dynes/cm}^2=4\times10^{-15}\ \text{atm.}\,, \tag{11}$$

where $N=5700$ atoms/cm^3; $k=$Boltzmann's constant. The mean pressures for the higher layers are correspondingly:
for

$$y<11.5\times10^8\,\text{cm};\quad 0.6\times10^{-15}\ \text{atm.}\,,$$

and for

$$y<\ 8.4\times10^8\,\text{cm};\quad 0.3\times10^{-15}\ \text{atm.}$$

If we further consider the pressures in the three layers into which the chromosphere is divided by our points of observation to be constant, the individual values, which may be regarded as the average pressure for these layers, can be estimated by means of the foregoing figures. Let us designate the thickness of these arbitrarily chosen layers by a, b, c, and the corresponding pressures by P_a, P_b, P_c. We then have the equations

$$\left.\begin{aligned} aP_a+bP_b+cP_c&=(a+b+c)\ p\,,\\ aP_a+bP_b\quad\ &=(a+b)\ \tfrac{1}{6}\ p\,,\\ aP_a\quad\ \ &=a\ \tfrac{1}{15}\ p\,. \end{aligned}\right\} \tag{12}$$

By solving these equations and inserting the numerical values we find the following stratification of pressure which is still affected with considerable uncertainty, but surely gives the right order of magnitude:

[1] The assumption of a smaller height of the chromosphere H would also improve the agreement between theory and observation. We shall not, however, make an attempt to press the data.

Layer	Thickness	Pressure	
a	8.4×10^{8} cm	$P_a\sim0.3\times10^{-15}$ atm.	Total height $=14\times10^{8}$ cm
b	3.1×10^{8} cm	$P_b\sim 2\times10^{-15}$ atm.	Average pressure $=4\times10^{-15}$ atm.
c	2.5×10^{8} cm	$P_c\sim 19\times10^{-15}$ atm.	

Height above the Photosphere	Ca^+ Pressure
0–2500 km	19×10^{-15} atm.
2500–5600	2×10^{-15}
5600–14,000 (?)	0.3×10^{-15}

In order to summarize all the results at this point it may be worth while to state once more that the average velocity obtained for the Ca^+ atoms is 15 km/sec. The thermal velocity of Ca^+ at $T=4800^\circ$ K is

$$v_{th}=\sqrt{\frac{3kT}{M}},$$

where $M=6.60\times10^{-23}$ is the mass of the Ca^+ atom. Hence $v_{th}=1.7$ km/sec.

6. COMPARISON WITH MILNE'S THEORY

Since a nearly isothermal atmosphere with a pressure of only a fraction of an atmosphere would immediately be compressed to a thickness of a few hundred kilometers under the action of the solar gravitation, E. A. Milne has worked out in a series of papers the hypothesis that the chromosphere is supported essentially by light-pressure. If this assumption is right, the radiation absorbed, or better, scattered away by the H_3 and K_3 lines, must be just sufficient to support the spectroscopically determined mass of the Ca^+ chromosphere. Measurement of the areas under the curves in Figure 6 leads to the result that the quantity of light scattered away by the H_3 and K_3 lines corresponds to the complete absorption of a band 0.3 Å wide having the intensity in the middle of the broad H and K lines. As this residual intensity, produced by the influence of collisions of Ca^+ atoms with other particles, is about 1/10 of the intensity of the continuous spectrum of the same spectral region, the H_3 and K_3 lines correspond to the complete absorption of a zone of the continuous spectrum 0.03 A wide.

According to Milne[1] the number of atoms which can be supported by a band of the breadth $\Delta\nu$ is given by

$$N_m = \frac{4\pi h\nu^3}{gMc^3} e^{-\frac{h\nu}{kT}} \Delta v \,, \tag{13}$$

where $g = 2.74 \times 10^4$ is the gravitation at the solar surface; $M = 6.6 \times 10^{-23}$ is the mass of Ca^+ atoms; and ν is the frequency of the Ca^+ lines. Substitution of the numerical values gives

$$N_m = 7 \times 10^{12} \text{ atoms.}$$

On the other hand, our spectroscopic determination gives

$$N_{sp} = NH = 5700 \times 1.4 \times 10^9 = 8 \times 10^{12} \text{ atoms.}$$

The two values agree therefore far within the accuracy of measurement. Milne himself has given in his publications pressures of $\sim 10^{-13}$ atm. for the chromosphere, while we find $\sim 10^{-15}$ atm. The origin of this difference is that he has overestimated the total absorption corresponding to the chromosphere.[2]

Our result has a further important consequence: It indicates that calcium cannot be doubly ionized to an appreciable degree in the chromosphere. For if it were, the light-pressure calculated from the total absorption of the H_3 and K_3 lines would not be sufficient to support the spectroscopically determined quantity of Ca^+ and at the same time the corresponding quantity of Ca^{++}.

On the other hand, according to Saha's formula, the ratio of Ca^{++}/Ca^+ at a temperature of $T = 4830° K$ and a partial pressure of 10^{-15} atm. for the Ca^+ atoms is given by

$$\log_{10} \frac{Ca^{++}}{Ca^+} (Ca^+ + 2Ca^{++}) = \log_{10} Ca^+ \left[\frac{Ca^{++}}{Ca^+} + 2 \left(\frac{Ca^{++}}{Ca^+} \right)^2 \right]$$
$$= -\frac{5036\, I}{T} + 2.5 \log_{10} T - 6.5 \,, \tag{14}$$

where the chemical symbols are put for the pressures in atmospheres, and $I = 11.82$ volts represents the ionization potential of Ca^+. One obtains, thus,

$$\frac{Ca^{++}}{Ca^+} \approx 350 \,.$$

[1] See *Monthly Notices, R.A.S.*, **88**, 188, 1928; esp. p. 192.

[2] Milne himself has remarked, however, that smaller figures would probably correspond more closely to reality. Cf. *op. cit.*, p. 193 n.

It follows that the degree of ionization is much lower than the calculation made on the assumption of thermal equilibrium would indicate.

7. DYNAMICS AND STABILITY OF THE CHROMOSPHERE[1]

We have still to discuss why the average velocity of the Ca^+ atoms in the chromosphere (15 km/sec.) is considerably higher than the thermal velocity corresponding to the solar temperature. This discussion will lead us to considerations of stability.[2]

The lower boundary of the chromosphere should be determined by the fact that at a certain pressure collisions between the different particles begin to play an important part (condition I).[3] On the other hand, however, according to Milne, the chromosphere can be supported only if the residual intensity in the middle of the Ca^+ lines[4] has a certain value. Because of this, according to equations (9) and (4), the numerical value of $NH/\Delta\lambda_0$ is fixed (condition II). Conditions I and II permit a separation of the values of NH and $\Delta\lambda_0$, and these lead to a knowledge of the whole chromosphere. At the present time, however, it is not quite possible to carry through these ideas quantitatively.

The necessity for a "turbulent" motion of the chromosphere of an average velocity corresponding to the Doppler width has another interesting consequence. Since the entropy of the chromosphere cannot increase, in course of time these motions will tend to become equal and approach the value ~ 1.7 km/sec., the thermal motion at the solar temperature. If, however, $\Delta\lambda_0$ becomes smaller than the assumed value, the total absorption will no longer be sufficient to support the chromosphere, which will therefore collapse. Then, however, the light-pressure will again expel atoms until the "normal state" is regained, when the play begins anew. In appearance, the chromosphere can never reach a stationary state, but will always be going up and down; in fact, spectroheliographic observations show that the chromosphere often looks like a "burning prairie." The single flames seem to change rather rapidly. Our explanation thus leads to a steady transition between the normal chromosphere and

[1] These considerations are merely a first attempt to explain the observations. Our very schematic ideas are evidently still too rough for giving a complete theory.

[2] Cf. esp. Milne, *Monthly Notices, R.A.S.*, **86**, 459, 1927.

[3] Cf. A. Unsöld, *loc. cit.*

[4] The reference here and in the following is to a mean value for H_3 and K_3.

the prominences which is very satisfactory from the viewpoint of spectroheliographic observations. The changes of brightness of the photosphere through changes in light-pressure will naturally also give rise to motions.

8. THE DISTRIBUTION OF PRESSURE IN THE CHROMOSPHERE

In a former paper[1] the author has proposed a theory of the chromosphere different from Milne's. However, as Milne[2] and Anderson[3] have shown, the assumption of constant light-pressure which was made in that paper was not justified. Moreover, as Milne has already assumed (and as was demonstrated in section 6), Saha's formula cannot be applied to the chromosphere, since the conditions deviate very much from thermal equilibrium.

Milne, on the other hand, has neglected in his theory the action of ionization. The dynamic equilibrium is regulated by changing the number of excited atoms. This assumption is justified, as may easily be seen, as long as the free path for recombination of Ca^{++} atoms with electrons is greater than the height of the chromosphere. According to an estimate by Milne,[4] this requirement is in general likely to be fulfilled. At very great heights it is possible that Ca^{++} atoms, a very small degree of ionization being sufficient, produce an apparent diminution of the coefficient $1-\mu=$light-pressure: gravitation, and hence a rather definite boundary for the chromosphere. The decrease of pressure with increasing height as derived in section 5 of the present paper, in so far as order of magnitude is concerned, is in agreement with Milne's theory. More cannot very well be expected in the present state of affairs.

In conclusion, I should like to express my heartiest thanks to Dr. St. John, Dr. Dunham, and Dr. Nicholson for much help and advice; and to Miss Louise Ware for her help in the photometric measurements. I am indebted furthermore to the International Education Board for giving me the opportunity to work at the Mount Wilson Observatory.

CARNEGIE INSTITUTION OF WASHINGTON
MOUNT WILSON OBSERVATORY
January 1929

[1] *Zeitschrift für Physik*, **44**, 793, sec. 7, 1927.
[2] *Op. cit.*, **88**, 188, 1928.
[3] *Zeitschrift für Physik*, **46**, 445, 1927.
[4] *Loc. cit.*, **88**, 188, sec. 7, 1928.

ON THE DISTRIBUTION OF LIGHT IN THE SOLAR CORONA

By H. T. STETSON and L. B. ANDREWS

ABSTRACT

The results of a photometric study of standardized coronal plates taken at the Sumatra eclipse of January 14, 1926, indicate a diminution of *brightness* in the solar corona which *cannot be expressed as a simple function* of the *radial distance* either from the limb or the sun's center. Exponential formulae, however, have been found which will represent, satisfactorily, the diminishing brightness with increasing distance from the solar center. These are as follows: For the region of 2.0–2.55 solar radii from the center, $L \propto \rho^{-7}$; from 2.55 to 3.1 radii, $L \propto \rho^{-4}$; from 3.1 to the limit of measurement, $L \propto \rho^{-2}$; where L equals the illumination seen from the earth, ρ = the radial distance from the sun's center in terms of the sun's radius as a unit. A *composite curve* is drawn, giving the combined results *of* observation at *several eclipses.*

Discrepancies in results of different investigations are largely due to the difference in the regions analyzed. There is *little evidence* that the *laws of distribution of brightness vary appreciably* at different eclipses.

A report on photometric measures of coronal brightness for the eclipse of January 14, 1926, has already been published.[1] The results of an analytical study of the distribution in brightness with increasing distance from the sun for the same eclipse are presented herewith.

Through the courtesy of Dr. Miller of the Swarthmore expedition, some of the plates taken with a 6-foot camera were transferred to the Harvard party for photometric standardization before development and subsequently loaned to the Harvard Astronomical Laboratory for measurement. The courtesy is especially appreciated since the Harvard expedition to Sumatra undertook no direct photography of the corona.

The plates were standardized in the dark room on the evening following the eclipse and just prior to development. For the standardization, a Hefner lamp was used in conjunction with a Chapman-Jones plate tester. Of the standardized plates available, one was carefully selected as best suited to the problem, showing maximum freedom from fog, homogeneity of background, and a desirable extent of the corona.

The plate was an Eastman 40 and had been exposed through a lens of 15-cm. aperture and 180-cm. focal length for five seconds during totality.

[1] *Astrophysical Journal*, **66**, 65, 1927.

Some three hundred points in the coronal image were selected for measurement of opacity with the thermo-electric photometer to which reference has been made in earlier publications.[1] The points selected were scattered about through a region of almost 10 cm. diameter in which the moon's image, 19 mm in diameter, was central.

Measures within a solar radius of the moon's limb were disregarded on account of extreme overexposure in this region. The re-

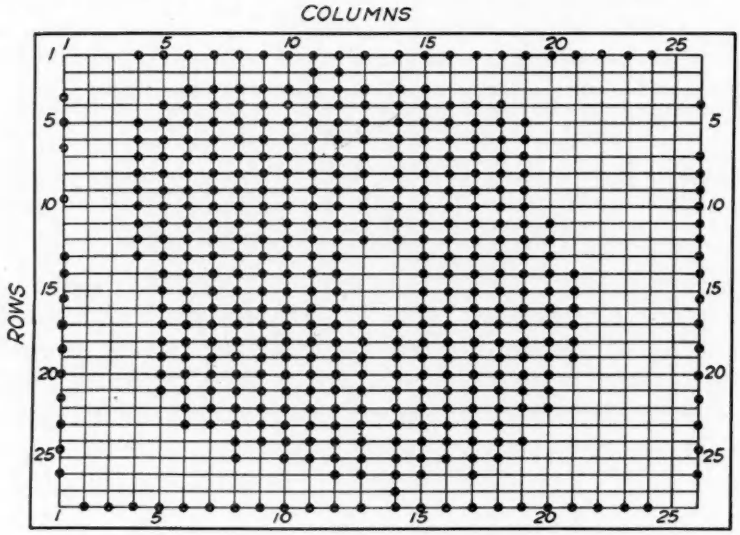


FIG. 1.—Diagram of measured points in the corona

maining measures ranged from about 2 to about 6 solar radii from the sun's center.

Space can hardly provide for the table of original measures. A system of rectangular co-ordinates for numbering points selected for the measures is shown in Figure 1. The circles indicate the measured points.

In interpreting galvanometer deflections in terms of magnitudes, and hence of brightness, it has been customary to use the form $D-D'/D$, where D = the deflection on the clear plate and D' = the deflection on the image. The resulting reduction curve, however, is

[1] *Ibid.*, **43**, 253, 1916; **58**, 36, 1923.

not very sensitive for the extreme regions where the density is very high or low. Miss Harwood and Professor King have suggested the form $\log D'/D$ for great densities.[1] In the present instance, the form $\log 1000\, D'/D$ was adopted in making the reductions, the factor 1000 being introduced for the purpose of producing a convenient scale for the ratio D'/D.

Inasmuch as the density of the plate was considerable, the relation was well suited to the purpose. For slight opacities the form $\log 1000\, (D-D')/D$ is suggested. For the reduction of plates involving considerable gradation one might split the reductions at the point where $D'=\frac{1}{2}D$, reducing those measures for which $D'<\frac{1}{2}D$ by the formula $\log 1000\, D'/D$, and those for which $D'>\frac{1}{2}D$ by the formula $\log 1000\, (D-D')/D$. The resulting curves are practically linear.

In applying the formula to the case in hand, sky fog necessitated obtaining the value of D from that portion of the plate which is protected during exposure by the lip of the plateholder. The measures taken at short intervals around the periphery of the plate indicated that the value of D was a decidedly varying quantity. A system of isophots was then established, which gave a satisfactory constancy of the sky values for widely different portions of the plate.

The reduction of D'/D into magnitudes and light-intensities was effected by the constants of the plate determined from the measurement of the standard squares in the usual manner.

Since the standard squares were impressed upon the sky-fogged background as was the image of the corona, the effect of fogging should be sensibly the same for the corona and the standard squares. Hence the plate gamma determined from the measures of the squares served for reducing the photometric measures of the coronal image into relative light-intensities without the introduction of additional corrections for fog.

The values of light-intensities deduced from these measures were then plotted against the corresponding distances from the sun's center taken in solar radii. While the curve was fairly determined from 3 solar radii outward, the points near the sun's limb were seen to scatter considerably.

It appeared that certain advantages might result through me-

[1] *Harvard Circular*, No. 312.

chanically integrating the coronal illumination by allowing the plate to rotate past the diaphragm of the photometer while the measures were being made. Accordingly, a rotary form of the thermo-electric photometer was constructed for completely remeasuring the plate by an independent method. In the apparatus the lamp-thermocouple system remained stationary while the stage on which the plate was mounted was rotated by an electric motor. The rotating stage could be moved in translation by a graduated screw. This made possible the integration of the light of the corona in zones at varying distances from the center of the sun.

To investigate the question as to whether or not measures obtained by such a method are directly comparable with those obtained by a stationary photometer, a series of test plates was made by exposing a rotary plate through a step shutter to a fixed source. Upon development each plate showed a series of concentric rings of diminishing density corresponding to a known difference in intensity. The relative opacities were then carefully measured on the rotary photometer at a variety of speeds ranging from 500 to 65 r.p.m. The same plates were then measured on the stationary photometer.

The results of a test for an Eastman 40 plate are given in Table I and indicate that the measurements obtained by the two methods are in good agreement.

In the rotary photometer the diameter of the aperture is 0.64 solar radii. On account of the large diameter of the aperture, a considerable portion of the corona is being intercepted at any particular setting. However, as the size of the aperture remains unaltered over the entire plate, the effect of its large diameter cannot change the general law of distribution of light in the corona but only serves to smooth the curve given by the integration of any particular zone. Furthermore, the effect of the varying aperture was tested observationally and the large aperture was found not to affect the general contour of the curve. The measures of opacity were interpreted as light-intensities from the measures of the standard squares as in the case of the stationary photometer. A comparison of the results of the two methods of measurement is shown in Figure 2.

TABLE I

ROTARY MEASURES OF EASTMAN KODAK 40 TEST PLATE

Speed R.P.M.	Ring No.	Galv. Read.	Zero Read.	Stand. Ref.	Galv. Defl.	Stand. Defl.
500	1	8.90	8.76	11.41	0.14	.....
	2	9.04	.....	.....	0.30	.....
	3	11.36	.....	.....	2.64	.....
	4	24.18	.....	.....	15.48	.....
	5	34.81	8.68	11.30	26.13	2.64
320	1	8.74	8.66	11.31	0.08	.....
	2	9.01	.....	.....	0.34	.....
	3	11.34	.....	.....	2.66	.....
	4	24.19	.....	.....	15.49	.....
	5	34.80	8.70	11.39	26.10	2.67
190	1	8.75	8.71	11.31	0.04	.....
	2	9.00	.....	.....	0.27	.....
	3	11.32	.....	.....	2.57	.....
	4	24.18	.....	.....	15.41	.....
	5	34.80	8.79	11.45	26.01	2.64
65	1	8.55	8.46	11.28	0.09	.....
	2	8.86	.....	.....	0.37	.....
	3	11.30	.....	.....	2.78	.....
	4	24.66	.....	.....	16.11	.....
	5	35.70	8.58	11.30	27.12	2.78
0	1	8.79	8.70	11.38	0.09	.....
	2	9.01	.....	.....	0.35	.....
	3	11.26	.....	.....	2.66	.....
	4	24.04	.....	.....	15.39	.....
	5	35.74	8.78	11.32	26.96	2.60

SUMMARY

Galvanometer Deflections

Ring	Speed				
	500	320	190	65	0
	Deflections				
1	0.14	0.08	0.04	0.08	0.09
2	0.30	0.34	0.27	0.35	0.35
3	2.64	2.66	2.57	2.63	2.66
4	15.48	15.49	15.41	15.41	15.39
5	26.13	26.10	26.01	26.33	26.96

In interpreting the results it was early apparent that a single law of light-distribution could not satisfactorily apply to the entire observed curve. An endeavor has therefore been made to fit the observed curve with as few formulae for light-distribution as possible.

For variations from the sun's limb as origin, it was found possible to fit the ends of the observed curve quite consistently by simple formulae, an inverse 3.7 power from 1.0 to 1.55 solar radii from the limb, and an inverse 1.5 power from 2.0 radii to the limit of observation. It was found necessary to use another formula for the region for 1.55–2.0 radii. This could be done by assuming an inverse 2.3 power. These formulae fit equally well the results exhibited in the curves made with the rotary and the stationary photometer. If we were to assume that the sun represents a continuum of diminishing density as we pass from the center to the outermost corona, then, from the point of view of physical significance, laws relating the variation in brightness to the distance from the sun's center might appear more valuable. Accordingly, an attempt was made to determine the laws based on the distance from the sun's center as origin. As a result we find an inverse seventh power to hold from 2.0 to 2.55 solar radii, an inverse fourth power from 2.55 to 3.1 radii, and an inverse square power from 3.1 to 5.0 radii.

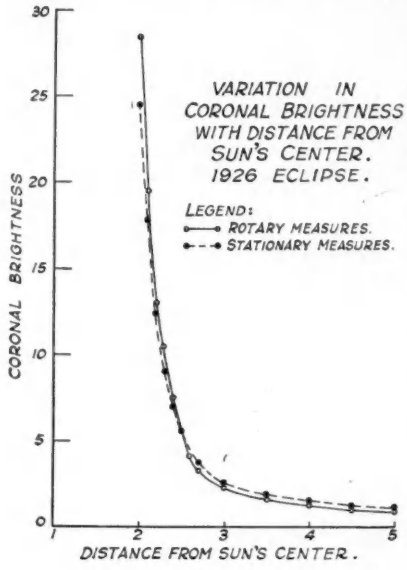


FIG. 2

Tables II and III show the agreement between the observed and computed values.

On investigation it is revealed that the differences in the formulae for distribution obtained by different observers is largely the result of the region investigated. The high inverse powers from the sun's center refer in general to the region within 1 solar radius of the limb,

as does also Becker's formula. The high power agrees closely with the law determined in this paper for the region 1.0–1.5 radii from the limb.

TABLE II

COMPARISON OF OBSERVED AND COMPUTED VALUES OF CORONAL BRIGHTNESS

Dist.*	Observed	Computed	From Limb
2.0	2.03	2.05	.....
2.2	1.00	1.00	.....
2.3	0.74	0.70	$d^{-3.7}$
2.4	0.54	0.53	.....
2.6	0.29	0.30	.....
2.6	2.56	1.98	.....
3.0	1.38	1.41	.....
3.5	1.00	1.00	$d^{-1.5}$
4.0	0.75	0.76	.....
4.5	0.63	0.59	.....
5.0	0.61	0.50	.....
2.6	1.46	1.32	.....
2.8	1.00	1.00	$d^{-2.3}$
3.0	0.79	0.79	.....

* Measured from the sun's center in solar radii.

TABLE III

COMPARISON OF OBSERVED AND COMPUTED VALUES OF CORONAL BRIGHTNESS

Dist.*	Observed	Computed	From Center
2.0	2.03	1.95	.....
2.1	1.38	1.38	.....
2.2	1.00	1.00	d^{-7}
2.3	0.74	0.73	.....
2.5	0.54	0.41	.....
2.7	0.29	0.24	.....
2.6	1.46	1.34	.....
2.7	1.18	1.16	.....
2.8	1.00	1.00	d^{-4}
2.9	0.89	0.87	.....
3.0	0.79	0.76	.....
3.1	1.33	1.27	.....
3.2	1.24	1.20	.....
3.3	1.15	1.12	d^{-2}
3.5	1.00	1.00	.....
4.0	0.75	0.77	.....
5.0	0.61	0.49	.....

* Measured from the sun's center in solar radii.

Table IV gives the various measures here discussed, and Figure 3 is a plot of these. The general agreement of measures in the region from the limb to 1 radius out from the limb is quite marked.

TABLE IV

	With Distances D Measured from the Sun's Center	
D^{-6}	H. H. Turner	*Monthly Notices, R.A.S.*, **61**, App., p. 4, 1900
D^{-6}	Pettit and Nicholson	*Pop. Ast.*, **14**, 548, 1906
D^{-8}	R. K. Young.....	*Astrophysical Journal*, **62**, 218, 1925
		Lick Obs. Bull., **6**, 166, 1911
	With Distances X Referred to the Sun's Limb	
X^{-2}	O. Bergstrand	*Etudes sur la Distribution de la Lumière dans la Couronne solaire*, Upsala, 1919
X^{-2}	M. Harwood and E. S. King	*Harvard Circular*, No. 312
	Becker Refers to an Arbitrary Origin and Expresses the Law As	
$(h+140)^{-4}$	L. Becker	*Phil. Trans. Roy. Soc.*, A, **207**, 1908

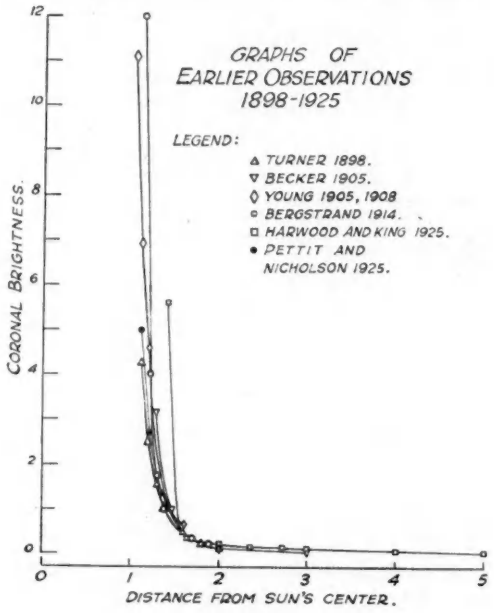


FIG. 3

By drawing a mean curve it would seem possible for us to extend the determinations in this paper to the inner portion of the corona and thus obtain a curve from near the limb to 4 radii from the limb. Of course, while it cannot positively be assumed that the same laws

TABLE V

Dist. Sun's Center (Solar Radii)	R. K. Young (1905)	R. K. Young (1908)	H. H. Turner (1898)	O. Bergstrand (1914)	Balanovsky + Perepelkin (1927)	Pettit + Nicholson (1925)	L. Becker (1905)	Harwood + King (1925)	Stetson + Andrews (1926)
1.035	.....	.....	.....	.....	29.00	.....	.....	.....	.....
1.050	.....	.....	.....	.....	21.50	.....	.....	.....	.....
1.075	7.96	7.25	.....	.....	14.30	.....	.....	.....	.....
1.1	6.59	7.41	4.26	16.00	10.00	4.74	.....	.....	.....
1.2	3.54	3.38	2.52	4.00	3.65	2.67	.....	.....	.....
1.3	1.88	1.67	1.56	1.78	1.65	1.65	1.89	.....	.....
1.4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.5	0.64	0.63	0.66	0.64	0.560	0.62	0.57	0.58	.....
1.6	0.49	0.38	0.45	0.45	0.350	0.41	0.36	0.42	.....
1.7	.....	.....	.31	.33	.230	.30	.246	.32	.....
1.8	.....	.....	.22	.25	.160	.22	.148	.26	.....
1.9	.....	.....	.16	.20	.130	.16	.107	.22	.....
2.0	.....	.....	0.12	0.16	0.08	0.12	0.082	0.18	0.126
2.1	.....	.....	.....	.....	0.06	0.103	0.062	0.16	0.087
2.2	.....	.....	.....	.....	.....	.079	.047	.12	.061
2.3	.....	.....	.....	.....	.....	.056	.035	.10	.045
2.4	.....	.....	.....	.....	.....	.040	.025	.086	.034
2.5	.....	.....	.....	.....	.....	0.030	0.020	0.064	0.025
2.6	.....	.....	.....	.....	.....	0.018	0.016	0.056	0.020
2.7	.....	.....	.....	.....	.....	0.009	.013	.051	.017
2.8	.....	.....	.....	.....	.....	.....	.012	.047	.015
2.9	.....	.....	.....	.....	.....	.....	.010	.043	.012
3.0	.....	.....	.....	.....	.....	.....	0.008	0.040	0.011
3.5	.....	.....	.....	.....	.....	.....	.....	0.026	.....
4.0	.....	.....	.....	.....	.....	.....	.....	.020	0.006
4.5	.....	.....	.....	.....	.....	.....	.....	.015	.....
5.0	.....	.....	.....	.....	.....	.....	.....	.011	0.005
5.5	.....	.....	.....	.....	.....	.....	.....	.008	.....
6.0	.....	.....	.....	.....	.....	.....	.....	0.006	.....

will hold from one corona to the next, yet as a sufficiently good first approximation this assumption seems justified. A summary for the expressions of coronal brightness with increasing distance from the sun as determined from the several investigations mentioned is shown in Table V.

The formula for the portion from the limb to 1 radius from the

limb is found to be an inverse seventh power in continuation of that found from 1.0 to 1.55 radii from the limb. A combined curve of coronal brightness with increasing distance from the sun based on all investigations to date is given in Figure 4.

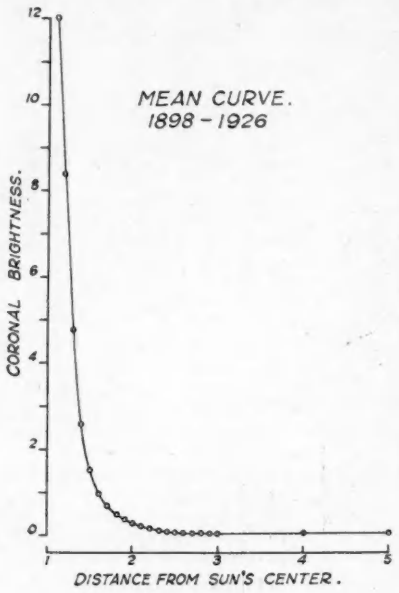


FIG. 4.—Composite curve of variation of coronal brightness drawn from observations of several eclipses.

SUMMARY

From measures of a standardized photograph, the variation of light in the corona has been found to follow an inverse seventh power of the distance from the center of the sun throughout the region from 2.0 to 2.55 radii; an inverse fourth power from 2.55 to 3.1 radii; and an inverse square law from 3.1 radii to the limits of measurement.

Considering the results of previous observers, a mean curve has been assumed for the region from the limb to 1 radius from the limb. This curve is also found to follow the inverse seventh power.

It seems fruitless to make any single law which will be applicable at widely varying distances from the limb, and it appears that many apparent discrepancies among previous eclipse observations are removed when due recognition is given to the various regions measured by different investigations.

HARVARD UNIVERSITY
December 1928

MINOR CONTRIBUTIONS AND NOTES

NOTE ON OTTO STRUVE'S PAPER ON THE STARK EFFECT IN STELLAR SPECTRA

ABSTRACT

The contours of the helium lines λ 4387 and λ 4472 have been determined in the stars 17 ζ Cassiopeiae, 88 γ Pegasi, and 85 ι Herculis in order to obtain quantitative data in regard to the presence of the Stark effect in stellar spectra. The line at λ 4470 *which Struve identifies as a forbidden line of helium is distinctly seen* in each of the contours of *He* 4472. Its position agrees well with that measured by Struve. The *ratios of the intensities* of λ 4470 to λ 4472 are 0.31, 0.32, and 0.23, respectively, for the foregoing stars. In each contour the line λ 4387 is unsymmetrical, indicating a component at wave-length 4386.8 I.A. and with the *ratio of its intensity* to that of λ 4387 equaling 0.21. On two of the spectrograms of 17 ζ Cassiopeiae the contours of *He* 4922 were obtainable. These have an asymmetry that would be given by a component at λ 4920.2 with a ratio of its intensity to that of λ 4922 of 0.38. The *data supports* the presence of the Stark effect in the spectra of these stars.

In the preceding paper Struve identifies the unknown line at λ 4470 as the forbidden line of helium. This line makes its appearance in strong electric fields. He also calls attention to the nonsymmetrical broadening of other helium lines in electrical fields. It seemed to the writer that quantitative data on the contours of the helium lines λ 4387 and λ 4472 would help in the establishment of the presence of the Stark effect in stellar spectra.

Accordingly, plates taken with the Bruce spectrograph with a dispersion of the three prisms were analyzed with the registering microphotometer. The results for three stars in which the component λ 4470 is strong are shown in Figure 1. The abscissae are distances in Angstroms from the position of maximum intensity of the line; the ordinates are percentages of absorption of the continuous spectrum. Each contour, excepting the second one of line λ 4387 in 17 ζ Cassiopeiae, is obtained without any smoothing of the small irregularities in the density-curves. Each spectrum definitely shows the 4470 line, and its contour is obtained by subtracting the intensities on the red side of line 4472 from the corresponding values on the violet side. The continuous curves in Figure 1 are these contours. They vary in shape but each shows a definite maximum. The position of these maxima vary from displacements of 1.52 to 1.83 A

from the center of *He* 4472. The mean displacement is 1.65 A, which corresponds to a wave-length of 4469.86 I.A. Struve's value is 4469.88 I.A.

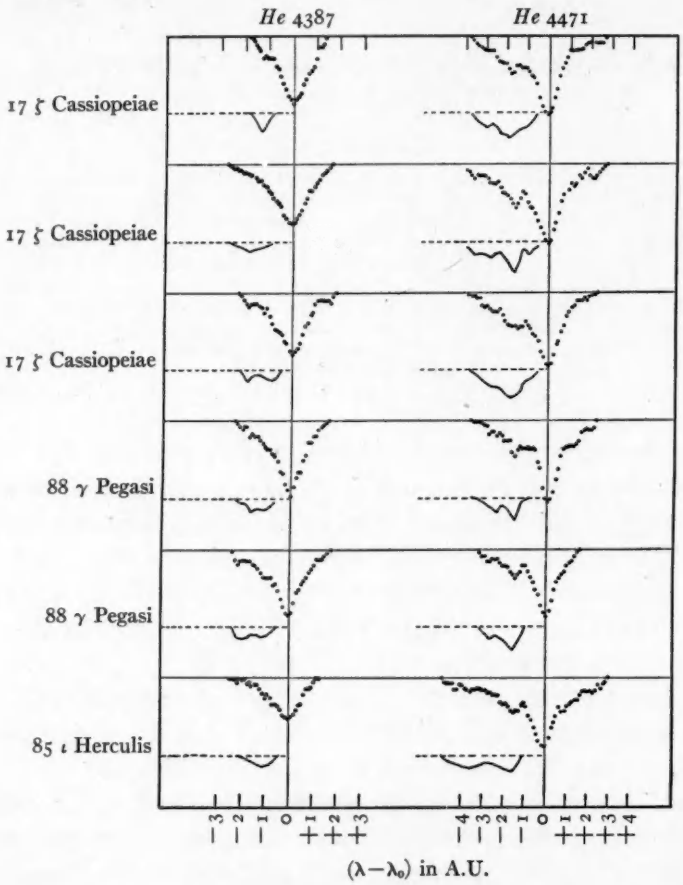


FIG. 1.—The contours of the helium lines λ 4387 and λ 4471 in the spectra of 17ζ Cassiopeiae, 88 γ Pegasi, and 85 ι Herculis, showing the forbidden line of helium λ 4470 and the asymmetry of λ 4387 due to the Stark effect.

The ratios of intensities of λ 4470 to λ 4472 are 0.31, 0.32, and 0.23, respectively, for 17 ζ Cassiopeiae (B3), 88 γ Pegasi (B2), and 85 ι Herculis (B3).

In each spectrum the line λ 4387 is seen to be asymmetrical, and on the assumption that the widening to the violet is due to a

component, its contours are obtained as in the case of λ 4470. The mean wave-length of the component is 4386.79 I.A., and the mean of the ratios of its intensities to those of λ 4387 is 0.21.

Struve predicts that the helium line 4922 should have a component at λ 4920, another forbidden line. This region of the spectrum is not in good focus, and is generally underexposed on plates taken with the Bruce spectrograph as it is normally used. Two plates of 17 ζ Cassiopeiae have a good *He* 4922, and a microphotometer tracing shows it to be distinctly asymmetrical to the violet side. This is not instrumental as the comparison lines are symmetrical. Obtaining the contour of the component as before, its wave-length is 4920.2 I.A. The ratio of its intensity to that of λ 4922 is 0.38.

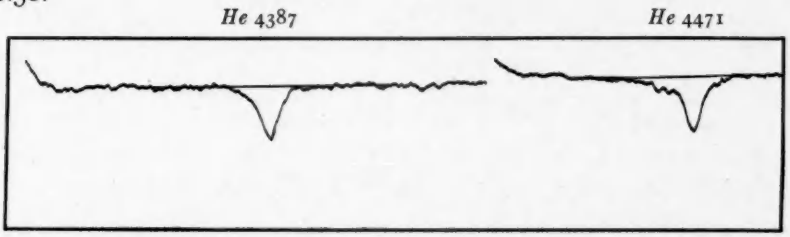


FIG. 2.—Density-curves of the helium lines λ 4387 and λ 4471 in the spectrum of 82 δ Ceti.

The star 82 δ Ceti shows the components quite prominently, but as the spectrograms were not standardized the contours are not determined. However, a density-curve of the lines is reproduced in Figure 2.

Each of the stars used has been announced as a spectroscopic binary. However, it seems very improbable that the components of the lines are due to the binary character of these stars. The constancy of the positions of the components in the six spectrograms of these three stars and of the thirteen spectrograms of the four stars in Struve's paper[1] is convincing evidence.

Accordingly, the asymmetries of the helium lines seem to point definitely to the presence of the Stark effect in these stars.

C. T. ELVEY

YERKES OBSERVATORY
March 1, 1929

[1] Otto Struve, *Astrophysical Journal*, **62**, 198, 1925.

NOTE ON THE RADIAL VELOCITY OF THE CLUSTER-TYPE VARIABLE RR LYRAE

ABSTRACT

The *elements* derived in *Mount Wilson Contribution*, No. 351, harmonize the epoch of light maximum and velocity minimum used in that investigation, but *do not* satisfactorily *represent later photometric data* which were overlooked. *All light maxima and velocity minima*, however, are cared for by a *formula* developed *by Prager. This formula is*, therefore, *to be used* in preference to my own.

From radial velocities obtained during 1911 and 1912 Kiess[1] found that maximum light and minimum velocity for the cluster-type variable RR Lyrae are practically coincident.

Radial velocities obtained mostly in 1926 and 1927 at the Mount Wilson Observatory, together with Kiess's values and the photometric data to 1922, gave the satisfactory formula:[2]

$$\left.\begin{matrix}\text{Max. light}\\ \text{Min. vel.}\end{matrix}\right\} = \text{J.D. } 2414856.480 + 0^d.56681729E + 0^d.121(10^{-4}E)^2 \text{ (G.M.A.T.).}$$

Unfortunately, as Hertzsprung has communicated to me by letter, important data on maximum light[3] were overlooked which extend the photometric material to 1926. Further, Prager[4] has given an excellent representation of all known light maxima in the formula:

$$\text{Max. light (G.M.A.T.).} = \text{J.D. } 2414856.4083 + 0^d.56683735E - 0^d.0693 \sin [0^\circ.0155(E-1200)] + 0^d.0086 \sin [0^\circ.0554(E-325)]$$

My formula does not represent the extended photometric data as well as Prager's; on the other hand, his harmonizes the two series of radial velocities with each other and with the phases of light maximum quite as satisfactorily as my own. At present, therefore, Prager's formula affords the best available means of representing the phase relations of light- and radial-velocity variations for RR Lyrae.

The conclusions given in *Contribution* No. 351 as regards the differences between Kiess's radial-velocity curve and my own are so little altered by assembling the velocities by means of Prager's

[1] *Lick Observatory Bulletins*, **7**, 140, 1913.

[2] *Mt. Wilson Contributions*, No. 351; *Astrophysical Journal*, **67**, 319, 1928.

[3] *Publications de l'Observatoire astronomique de Tachkent*, No. 2, 1927; *Veröffentlichungen der Universitäts-Sternwarte zu Berlin-Babelsberg*, **5**, Heft 4, 1926.

[4] *Op. cit.*, p. 23.

formula that they need scarcely be commented upon. The one large difference—that in the amplitude of velocity variation—cannot of course be greatly changed. A definitive determination of these differences will require a much larger number of radial velocities. Some have been obtained during 1928, and it is hoped that others may be added each season.

ROSCOE F. SANFORD

CARNEGIE INSTITUTION OF WASHINGTON
MOUNT WILSON OBSERVATORY
January 1929

REVIEWS

Astrophysique. By JEAN BOSLER. *Cours d'Astronomie*, Vol. III. Paris: Librairie Scientifique Hermann et Cie., 1928. 8vo. Pp. ii+723. Plates 47, figs. 205. Fr. 140, unbound.

The book under review is a companion volume to the well-known *Theoretical Astronomy*, by H. Andoyer (3d ed., 1923), and the *Practical Astronomy*, by H. Andoyer and A. Lambert (2d ed., 1923).

The lectures delivered at the Sorbonne, in 1919–1922, by J. Bosler, now director of the observatory and professor of the University of Marseilles, form the nucleus of this book. As a concluding third volume of the *Cours d'Astronomie* de la Faculté des Sciences de Paris it does not attempt to be more up to date than the last editions of the two volumes by Andoyer. It is to be regretted, however, that this best French work on astrophysics should be about four years old at the time of its publication.

The book is divided into four parts: instruments and methods (115 pp.); the sun (170 pp.); planets, comets, and meteors (200 pp.); and stars and nebulae (210 pp.).

The first part gives a very readable and rounded-out account on the ways and means of astrophysical research. The physics of the solar system receive a rather detailed treatment in the two central parts of the book; a condensation, especially of Part III, would be desirable, with a better balance of the whole structure in view. The last part, dealing with stars and nebulae, suffers from inadequate allotment of space and from the neglect of important recent data available at the time of publication of the volume, and which have found their way into most modern textbooks on astrophysics.

The treatment is essentially descriptive, the use of mathematics being reduced to a minimum. The author's logical way of thinking is reflected in the clear and simple style so suitable for a textbook.

In addition to an analytical Table of Contents (12 pp.) there is an Alphabetical Index of subject matter (14 pp.), an exhaustive Name Index (8 pp.), and a list of the forty-seven plates (3 pp.).

Some of the illustrations—figures in the text as well as plates—would gain if reproduced from better photographs. The proofreading was done as carefully as the indexing.

The volume will probably be found very valuable, in French-speaking countries, as a text- and reference-book. A. POGO

Trattato di Astronomia Siderale. Vol. I: *Parte generale.* By GIUSEPPE ARMELLINI. Bologna: Nicola Zanichelli, 1928. 8vo. Pp. xi+393. Star charts 2, figs. 82. Lire 85, unbound.

In the nineteenth century several excellent treatises on what is tautologically called "sidereal" or "stellar" astronomy have been written. In the last few years several excellent works on astrophysics or even on cosmic physics (*Physik des Kosmos*) have surveyed the rapid progress of research in solar and stellar physics, stellar statistics, cosmogony, and related fields.

And now comes the disappointing first volume of a one-man treatise on what the author chooses to call "stellar astronomy." Here are the plans for this ambitious project:

Volume I: Description of the sky. Methods of observation and of measurement used in stellar astronomy.

Volume II: Results of observations. Modern research concerning the life and evolution of the stars, and their internal constitution. Multiple and variable stars.

Volume III: Stellar statistics. Motions of the stars and especially of the sun. Nebulae. Clusters. General constitution of the stellar universe.

In the Preface to his treatise, the director of the astronomical observatory and professor of the Royal University of Rome points out that "stellar astronomy is the most modern chapter of the Science of the Heavens" (because it dates only from the end of the eighteenth century). To him, apparently, astrophysics is a mere paragraph in this "most modern chapter," an *ancilla astronomiae.* This attitude is probably responsible for the generous allotment of space to slightly or considerably obsolete chapters the non-astrophysical nature of which is above suspicion.

To avoid the *esposizione popolare e senza calcoli* of recent *manuali di divulgazione*—in which class he includes, e.g., the revision of Young's *Astronomy* by Russell-Dugan-Stewart—the author decided to combine "a simple and synthetic form with a rigorously scientific method"; but he relegates, nevertheless, whatever higher mathematics he offers, to the footnotes, in order to make his treatise accessible not only to "young astronomers" but also to a wider circle of readers.

The present first volume of the treatise consists of two parts: "Uranografia" and "Uranometria."

The first two chapters of the "Uranografia" are of a purely descriptive nature, of course, and deal—on scores of pages—with the nomenclature of stars and constellations and their relative positions in the sky. Two pages are devoted to the history of celestial globes, from Eudoxos (409–356 B.C.) to the perforated hollow 350-cm globe of Olearius (1654)

and the 385-cm globe of Cornelli (1685); of course, the planetarium of Zeiss is not mentioned. The third and last chapter of more than a dozen pages is devoted to the *difficile problema* of dynamic, chromatic, and parallactic scintillation (twinkling); the history of the problem from Aristotle to Arago (1852) and Exner (1881) is retraced step by step, the contributions of Donati (1865), Mossotti (1865), Montigny (1856), and Respighi (1865) getting especial attention. Three pages of this chapter grafted on the "Uranografia" are given over to scintillometers; in the second part of the volume, dealing with "methods of observation and of measurement used in stellar astronomy," the reader is laconically (p. 308) referred to treatises on general astronomy for the description of the incidentally mentioned "common position micrometer."

The second part of the volume, the "Uranometria," consists of five chapters: stellar photometry; stellar spectroscopy; interferometry and stellar diameters; parallaxes and proper motions; and, finally, stellar catalogues. The author has succeeded in presenting clearly the subject matter selected by him for the first four chapters of this "Uranometria." The last chapter suffers, in common with the other bibliographical references in this volume, under a strongly personal bias. A treatise in three volumes can be expected to give a more generous, representative, and up-to-date bibliography in its special field. Readers who are supposed to need information as to how to find Polaris by the "pointers" of Ursa Major should not be told of the existence of catalogues of double stars by Flammarion—or even by Secchi—if the names of but a chosen few (p. 376) are revealed to them, and if Aitken is introduced, in a footnote, as the man responsible for some "Measures of a Selected List of Double Stars" (*Lick Observatory Bulletins*, 1924; no volume, page, or bulletin number, of course).

There is no Name or Subject Index; neither is there a promise, in the Preface, to the effect that the third volume will contain an Index—if, when, and as completed.

An Appendix contains a catalogue of 155 stars down to the third magnitude, and two maps show the stars of the first four magnitudes of the southern and of the northern hemispheres.

The list giving twelve errata is decidedly too optimistic. Neglecting inverted gammas posing as lambdas, and overlooking an incredible carelessness in the use of italics, quotation marks, and abbreviations in the footnotes, there is a generous sprinkling of usually rather harmless misprints.

The book is printed on good paper, but the halftones are not always successfully reproduced.

A. Pogo